Splitting solution scheme for material point method

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Abstract

Material point method (MPM) is a numerical tool which was originally used for modelling large deformations of solid mechanics problems. Due to the particle based spatial discretisation, MPM is naturally capable of handling large mass movements together with topological changes. Further, the Lagrangian particles in MPM allow an easy implementation of history dependent materials.

MPM has been successfully applied in many engineering applications. So far, however, research on MPM has been mostly restricted to explicit dynamic formulations with linear approximation functions. This is because of the simplicity and the low computational cost of such explicit algorithms. Particularly in MPM analysis of geomechanics problems, a considerable attention is given to the standard explicit formulation to model dynamic large deformations of geomaterials. Nonetheless, several limitations exist. In the limit of incompressibility, a significantly small time step is required to ensure the stability of the explicit formulation. Time step size restriction is also present in low permeability cases in porous media analysis. Spurious pressure oscillations are another numerical instability present in nearly incompressible flow behaviours.

This research considers an implicit treatment of the pressure in MPM algorithm to simulate material incompressibility. The coupled velocity ($\mathbf{v}$)-pressure (p) governing equations are solved by applying Chorin's projection method which exhibits an inherent pressure stability. Hence, linear finite elements can be used in the MPM solver. The main purpose of this new MPM formulation is to mitigate artificial pressure oscillations and time step restrictions present in the explicit MPM approach. First, a single phase MPM solver is applied to free surface incompressible fluid flow problems. Numerical results show a better approximation of the pressure field compared to the results obtained from the explicit MPM. The proposed formulation is then extended to model fully saturated porous materials with incompressible constituents. A solid velocity($\mathbf{v}_S$)-fluid velocity ($\mathbf{v}_F$)-pore pressure (p) formulation is presented within the framework of mixture theory. Comparing the numerical results for the one-dimensional consolidation problem shows that the proposed incompressible MPM algorithm provides a stable and accurate pore pressure field even without implementing damping in the solver. An accurate computation of pressure is important for modelling

volume preserving constitutive relationships. Finally, the coupled MPM is used to solve a two-dimensional wave propagation problem and a plain strain consolidation problem. One of the important features of the proposed hydro mechanical coupled MPM formulation is that the time step size is not dependent on the incompressibility and the permeability of the porous medium.