Technical Note

Stiffness of Clays and Silts: Modeling Considerations

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Abstract: A large database has recently been published that details the development of new empirical expressions for the stiffness reduction with strain of clays and silts. In this note, the same database is used to examine two major considerations for engineers using these expressions in numerical analyses: the transformation from secant to tangent stiffness and the effect of stress history. **DOI:** 10.1061/(ASCE)GT.1943-5606.0001104. *© 2014 American Society of Civil Engineers.*

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Introduction

The estimation and measurement of soil modulus reduction with increasing strain has been the subject of much research in geotechnical engineering (e.g., Kondner 1963; Hardin and Drnevich 1972a, b; Vucetic and Dobry 1991; Fahey 1992; Fahey and Carter 1993; Stokoe et al. 1994, 1999; Hardin and Kalinski 2005; Gasparre et al. 2007; Oztoprak and Bolton 2013; Wichtmann and Triantafyllidis 2013a, b). The importance of understanding small-strains for geotechnical design has been discussed extensively in Burland (1989) and Atkinson (2000).

Vardanega and Bolton (2013) have recently published a large database that was used to derive simple empirical expressions for modulus reduction for clays and silts. The substantive details of the database formulation, the sources of data, and their subsequent analysis will not be repeated here. Fig. 1 shows the Casagrande plot (Casagrande 1947) for the soils in the database: a variety of fine-grained soil types are represented.

Static and Dynamic Adjustments

The stiffness of fine-grained soils is well known to be rate-sensitive (e.g., Richardson and Whitman 1963). Vardanega and Bolton (2013) presented calibrated empirical expressions [based on the general form adopted in Darendeli (2001)] demonstrating that rate-effect adjustments are necessary when comparing data tested in different apparatuses. The new curves were compared with those of Vucetic and Dobry (1991), which do not explicitly account for rate effects, and which are now seen to be too widely spaced.

The database presented in Vardanega and Bolton (2013) had the original test data from 10 publications (67 tests) adjusted for rate effects to two representative strain rates, namely 10^{-6}/s and 10^{-2}/s, with the former attempting to simulate a standard triaxial test and the latter simulating a standard earthquake. This adjustment was based on the assumption of a stiffness variation of 5% per factor 10 on strain rate, providing an indication of the increase in stiffness that is implied when moving from 10^{-6}/s (static adjustment) to 10^{-2}/s (dynamic adjustment) in these two design situations.

Calibrated Stiffness Reduction Functions

The newly calibrated functions to describe the modulus reduction of clays and silts from Vardanega and Bolton (2013), and the prediction of the reference strain parameter (γ_{ref}) are as follows, for the database with the static adjustment applied:

$$\frac{G}{G_{max}} = \frac{1}{1 + \left(\frac{\gamma}{\gamma_{ref}}\right)^{0.74}} \tag{1a}$$

$$\gamma_{ref} = 2.2\left(I_p/1{,}000\right) \tag{1b}$$

where I_p is expressed numerically and not as a percentage.

For the database with the dynamic adjustment applied

$$\frac{G}{G_{max}} = \frac{1}{1 + \left(\frac{\gamma}{\gamma_{ref}}\right)^{0.94}} \tag{2a}$$

$$\gamma_{ref} = 3.7\left(I_p/1{,}000\right) \tag{2b}$$

where, again, I_p is expressed numerically and not as a percentage.

In this note, the same database is used to examine two major considerations for engineers using these expressions in numerical analyses:

1. The transformation from secant to tangent stiffness; and
2. The effect of stress history.

Small Strain Region

The reduction of the shear stiffness of a soil with increasing strain from its purely elastic maximum value G_{max} is sketched in Fig. 2 for both monotonic and cyclic tests. Referring to Figs. 2 and 3, one can say that $G_{max} = G_{sec} = G_{tan}$ in the linear elastic strain range and that at greater strains one may describe the modulus either as a secant (G_{sec}) or a tangent (G_{tan}). The use of G_{sec} rather than G_{tan} is preferred in the processing of test data because it is an order-of-magnitude less influenced by random errors (noise). Nevertheless, G_{tan} is

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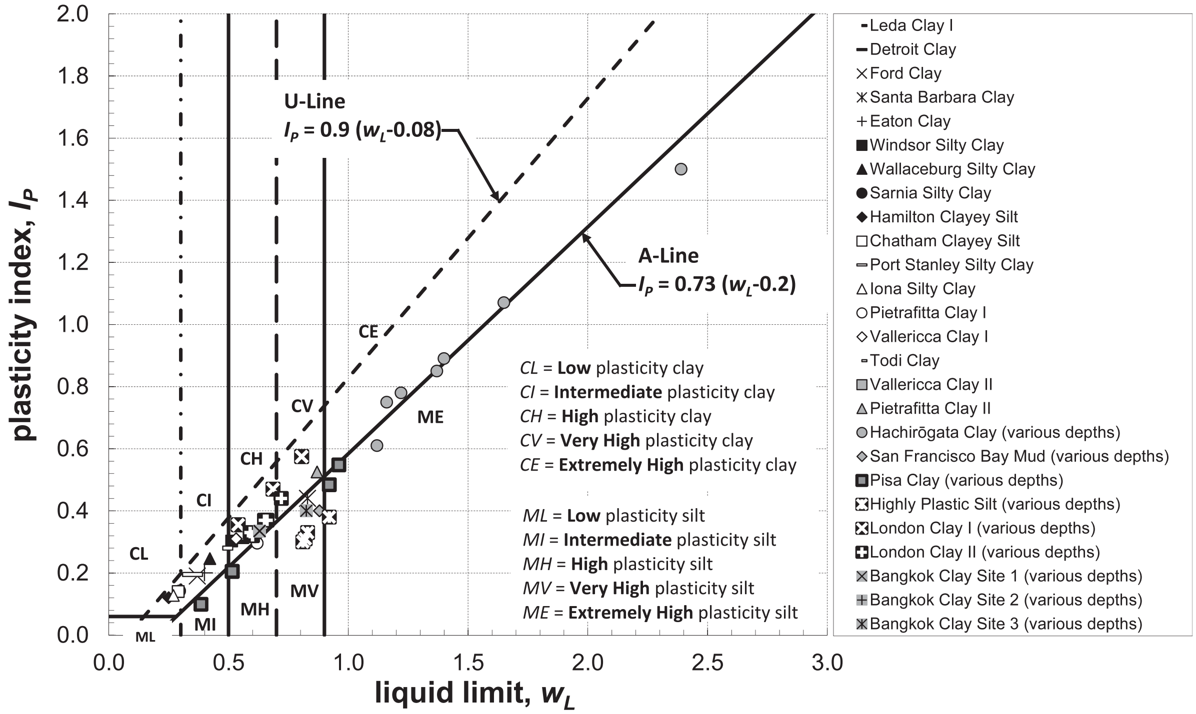

Fig. 1. Casagrande plot of the soils in the database presented in Vardanega and Bolton (2013) (chart design adapted from Casagrande 1947; Howard 1984; and BS5930 British Standards Institution 1999)

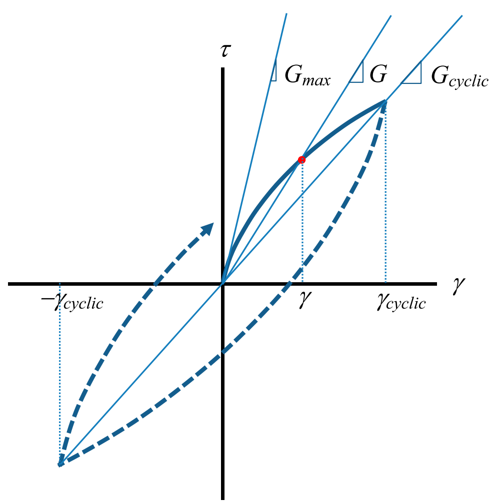

Fig. 2. Definitions of secant stiffness G, $G_{\max}$, and G_{cyclic}

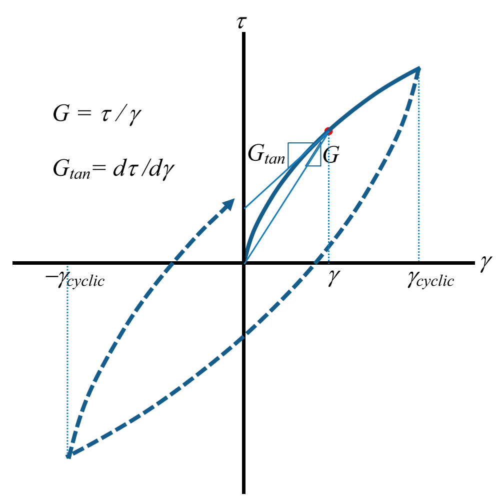

Fig. 3. Definition of tangent stiffness $G_{\tan}$

preferred in numerical procedures that require the assembly of an incremental stiffness matrix.

Tangent Stiffness

If the tangent stiffness is desired for numerical analysis, then it can easily be calculated from the secant stiffnesses that are quoted by Vardanega and Bolton (2013), which will consistently be referred to here simply as G. Given that Eqs. (1a) and (2a) have the same form [the form used in Darendeli (2001)], one can write

$$\frac{G}{G_{\max}} = \frac{1}{1 + \left(\frac{\gamma}{\gamma_{\text{ref}}}\right)^{\alpha}} \tag{3}$$

By definition

$$\tau = G\gamma \tag{4}$$

Differentiating Eq. (4) with respect to strain

$$G_{\tan} = \frac{d\tau}{d\gamma} = G + \gamma \frac{dG}{d\gamma} \tag{5}$$

By differentiating Eq. (3)

$$\frac{dG}{d\gamma} = -G_{\max}\frac{\alpha}{\gamma}\left(\frac{\gamma}{\gamma_{\text{ref}}}\right)^{\alpha}\frac{1}{\left[1+\left(\frac{\gamma}{\gamma_{\text{ref}}}\right)^{\alpha}\right]^{2}} \quad (6)$$

Substituting Eq. (6) in Eq. (5) and reorganizing, one obtains

$$\frac{G_{\tan}}{G} = 1 - \frac{\alpha}{\left[\left(\frac{\gamma_{\text{ref}}}{\gamma}\right)^{\alpha}+1\right]} \quad (7)$$

From Eq. (7) it can be seen that when $\alpha = 0.74$ (static adjustment)

$$\gamma = 0 \quad G_{\tan} = G_{\max} = G \quad (8a)$$

$$\gamma = \gamma_{\text{ref}} \quad G_{\tan} = G[1-(\alpha/2)] = 0.63\,G \quad (8b)$$

$$\gamma = 10\gamma_{\text{ref}} \quad G_{\tan} = G\{1-[\alpha/(1+0.1^{\alpha})]\} = 0.37\,G \quad (8c)$$

From Eq. (7) it can be seen that when $\alpha = 0.94$ (dynamic adjustment)

$$\gamma = 0 \quad G_{\tan} = G_{\max} = G \quad (9a)$$

$$\gamma = \gamma_{\text{ref}} \quad G_{\tan} = G[1-(\alpha/2)] = 0.53\,G \quad (9b)$$

$$\gamma = 10\gamma_{\text{ref}} \quad G_{\tan} = G\{1-[\alpha/(1+0.1^{\alpha})]\} = 0.16\,G \quad (9c)$$

Larger values of α produce a faster diminution in G with strain through Eq. (3), and even more so in $G_{\tan}$ through Eq. (7).

Consideration of Stress History

Database Variability

Table 1 presents the 67 tests that comprised the database presented in Vardanega and Bolton (2013) on 21 clays and silts reclassified according to their stress history. Twenty-four of the tests were on soils that were able to be classified as normally or lightly overconsolidated [overconsolidation ratios (OCR) $< \approx 2$]. Twenty-six of the tests were on soils that were able to be classified as heavily overconsolidated (OCR $> \approx 2$). Seventeen of the tests could not be classified in either category. [In the case of the data from Anderson and Richart (1976), insufficient information was provided about the natural soil deposits. In the case of the data from Kim and Novak (1981), there was apparently no attempt to replicate in situ conditions for the tests studied.]

Table 2 shows that the difference between the average curvature parameters for the three classifications is very small. This trend holds both for the database with the static adjustment and with the dynamic adjustment applied. Table 2 also demonstrates that the average value of the reference strain is not greatly different between the normally and lightly overconsolidated category and the heavily overconsolidated category. Vardanega and Bolton (2013), following the work of Vucetic and Dobry (1991), showed that γ_{ref} is a strong function of plasticity index. The static and dynamic adjustments also show that rate effects will have a significant effect on the reference strain. However, it would now appear that there is no significant influence of OCR on the reference strain. Fig. 4 shows Eq. (3) plotted with the average value of α_{stat} for the whole database denoted as α_{stat}(average), which is also plotted in Eq. (3) with values of α_{stat} ± 1 SD, denoted as α_{stat}(plus 1 SD) and α_{stat}(minus 1 SD), respectively. Additionally plotted is Eq. (3) with the average α_{stat} values shown in Table 2 for the normal and lightly overconsolidated classified soils and the heavily overconsolidated soils, denoted as α_{stat}(OCR < 2) and α_{stat}(OCR > 2), respectively. The upper and lower bounds of the normalized database presented in Vardanega and Bolton (2013) are also shown.

The influence of OCR on the curvature parameter (α) does not appear to be significant, simply from a visual inspection of Fig. 4. Similar trends are found using the database when the dynamic adjustment is applied.

It might be noted that the values of the average curvature parameters for the whole database are very similar to the average α-values used in Eqs. (1*a*) and (2*a*), but they are not identical because the number of available data points varies between the individual test curves. The selection of the best-fit regression line to determine the α-value ensures the maximum reduction of scatter.

Kinematic Yielding

The apparently marginal difference between lightly and heavily overconsolidated clays, in regard to their normalized stress-strain curves, deserves further comment. Fig. 5 is based on the kinematic yielding model of Jardine (1992) and Smith et al. (1992). Normally consolidated soil in situ can be represented by a point such as O in Fig. 5, standing on some plastic yield surface labeled Y_3. Outward-directed stress paths would cause plastic hardening and would create positive excess pore pressures in undrained tests. Inward-directed stress paths, such as those involved in field sampling and core extrusion in the laboratory, would initially involve linear and then nonlinear strains as the Y_2 yield surface is dragged down toward the p' axis. The location of the Y_3 yield surface may, however, cause the unloading stress path to create some irrecoverable hardening before the p' axis is reached. The state of isotropic stress at the outset of a standard triaxial compression test on a sample core may therefore be some point such as A in Fig. 5, consistent with a new Y_3 yield surface marked *disturbed* on the diagram. The fine-grained soils reported in the database as being normally consolidated in situ will generally have been tested in shear after isotropic relaxation to a point such as A. If the sample is isotropically overconsolidated from A it will achieve some point B prior to the shear phase of the test, as will clays which are naturally overconsolidated in situ.

An undrained triaxial compression test from either A or B will initially involve the same process of kinematic yielding at constant p' inside the Y_3 yield surface. This is represented by the dragging upwards of the Y_2 yield surface from points A or B, as shown in Fig. 5. According to Jardine (1992), both stress paths should begin with similar stress-strain relations consistent with a kinematic hardening rule. Eq. (3) can be regarded as an empirical expression of this proposition. If a constitutive modeler wished to propose that kinematic hardening be described by a unique expression, irrespective of stress history, then single values would be required of exponent α in Eq. (3) and a constant coefficient (i.e., the J-value) linking reference strain (γ_{ref}) and plasticity index (I_p) in Eqs. (1*b*) and (2*b*) for the strain-rate of interest.

At larger strains the influence of OCR has been shown to be significant (e.g., Vardanega et al. 2012). The findings of this note pertain to the small strain region.

Summary Remarks

The following summary points are made based on the work described in this note:

1. When performing numerical analysis the secant stiffness shear strain functions can be easily converted to tangent stiffness expressions: the curvature parameter (α) is directly linked to the diminishing stiffness with increased strain, even more so in tangent stiffness expressions.
2. Considering the fine-grained soils that could be classified as either normally or lightly overconsolidated and comparing

Table 1. Stress History Categorization of the Database Presented in Vardanega and Bolton (2013) along with Average Values of the Curvature Parameter (α) and Reference Strain (γ_{ref})

Publication	Soils tested	Number of tests	Average α_{stat}	Average α_{dyn}	Average $\gamma_{ref,stat}$	Average $\gamma_{ref,dyn}$	Notes on overconsolidation ratio	Classification of the soil deposit based on overconsolidation ratio
Anderson and Richart (1976)	Leda clay, Detroit clay, Ford clay, Santa Barbara clay, and Eaton clay	5	0.65	0.96	0.00065	0.0012	Insufficient information available	Unclassified
Kim and Novak (1981)	Seven Ontario fine-grained soils (low plasticity)	12	0.82	1.25	0.00036	0.00057	Natural OCR ranges from 1.8 to 6.8 Testing done at confining stresses $\gg p'$ in situ	Unclassified
Georgiannou et al. (1991)	Pietrafitta, Vallericca, and Todi clay	6	0.74	1.33	0.00065	0.00099	Authors state that the clays are overconsolidated; probably heavily overconsolidated, given that the natural condition of the clays are likely to be similar to those studied by Rampello and Silvestri (1993)	Heavily overconsolidated
Rampello and Silvestri (1993)	Pietrafitta and Vallericca clay	4	0.69	1.27	0.00062	0.00085	OCR values ~4.0 and 4.4	Heavily overconsolidated
Shibuya and Mitachi (1994)	Hachirōgata clay	7	0.65	1.07	0.0021	0.0036	The authors stated that the clay deposit was not believed to have been subjected to mechanical overconsolidation	Normally consolidated
Soga (1994)	San Francisco Bay mud (3- and 5-m-deep samples)	3	0.57	0.76	0.0012	0.0024	OCR ~1.5 at 5 m depth	Lightly overconsolidated
	Pisa clay (Horizon A)	1	0.71	0.93	0.00039	0.00078	OCR ~4.5 (4 m sample)	Heavily overconsolidated
	Pisa clay (Horizon B)	4	0.74	0.91	0.00068	0.0012	OCR average ~1.3 (varies from 1.2 to 1.5) (sampling from 10 to 19 m)	Lightly overconsolidated
Doroudian and Vucetic (1999)	Highly plastic Santa Barbara silt	4	0.82	1.13	0.00088	0.0017	OCR = 17 at 31.0-m depth (sampling from 9.5 to 64.6 m)	Heavily overconsolidated
Yimsiri (2001)	London clay	6	0.74	0.87	0.0013	0.0024	Ancient Eocene clay overlain by ~360 m of submerged sediments (Bishop et al. 1965) in the Wraysbury district (Hight et al. 2007); Chandler (2000) using geological evidence concluded that overburden removed ~200 m	Heavily overconsolidated

Table 1. (*Continued.*)

Publication	Soils tested	Number of tests	Average α_{stat}	Average α_{dyn}	Average $\gamma_{ref,stat}$	Average $\gamma_{ref,dyn}$	Notes on overconsolidation ratio	Classification of the soil deposit based on overconsolidation ratio
Teachavorasinskun et al. (2002)	Bangkok clay	10	0.77	0.88	0.00090	0.0016	No specific OCR details given in the note; however, Tanaka et al. (2001) give a value of 1.3 and Sambhamdharaksa et al. (2003) give values of OCR ~1.5−2.1 for the depth range 8.5–4 m (similar depth to the data studied)	Lightly overconsolidated
Gasparre (2005)	London clay	5	0.91	0.98	0.0019	0.0028	See description of London clay already given in this table	Heavily overconsolidated

Note: OCR = overconsolidation ratio.

them with the more heavily overconsolidated soils, it has been demonstrated that the normalized stress-strain curves of these two categories of geological materials may be quite similar in tests starting from a condition of isotropic effective stress. This has been explained as being indicative of a kinematic hardening function that is relatively insensitive to the initial mean effective stress within the state boundary surface (the Y3 yield surface), at least in the small-strain region which is the focus of this note. It must be remembered, of course, that the in situ stress state will in general have an effective stress ratio $K_0 \neq 1$, and that geotechnical processes in the field will generally involve more diverse stress paths than, for instance, simple triaxial compression, copious data of which are uniquely available in the literature. The influence of K_0 and of stress-path (in other words, the influence of anisotropy) on the shapes of stress-strain curves lies outside the scope of this note.

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Notation

The following symbols are used in this paper:

G = secant shear stiffness (see also G_{sec});
G_{cyclic} = secant shear stiffness measured in a cyclic test;
G_{max} = shear stiffness at very small strains (sometimes referred to as G_0);
G_{sec} = secant shear stiffness (see also G);
G_{tan} = tangent shear stiffness;
I_p = plasticity index;
K_0 = coefficient of earth pressure at rest;
p' = mean effective stress;
q = deviator stress;
w_L = liquid limit;
α = curvature parameter in the modified hyperbolic equation;
α_{dyn} = curvature parameter obtained when the fitting function is applied to data that had the dynamic adjustment applied (described in Vardanega and Bolton 2013);
α_{stat} = curvature parameter obtained when the fitting function is applied to data that had the static adjustment applied (described in Vardanega and Bolton 2013);
γ = shear strain;
γ_{cyclic} = shear strain amplitude measured in a cyclic test;
γ_{ref} = reference strain equal to the shear strain at 0.5 G_{max};
$\gamma_{ref,dyn}$ = reference strain for a test (or series of tests) where the data had the dynamic adjustment applied as described in Vardanega and Bolton (2013) to account for rate effects;
$\gamma_{ref,stat}$ = reference strain for a test (or series of tests) where the data had the static adjustment applied as described in Vardanega and Bolton (2013) to account for rate effects; and
τ = shear stress.

Table 2. Summary of Average α-Values and γ_{ref} Values for the Three Stress History Categories

Classification based on overconsolidation ratio	Number of tests in category	Average α_{stat}	Average α_{dyn}	Average $\gamma_{ref,stat}$	Average $\gamma_{ref,dyn}$
Normally consolidated and lightly overconsolidated soils	24	0.70	0.93	0.0013	0.0022
Heavily overconsolidated soils	26	0.77	1.10	0.0011	0.0017
Unclassified soils	17	0.77	1.17	0.00045[a]	0.00074[a]
All tests	67	0.75[b]	1.06	0.00097	0.0017

[a]Low average γ_{ref} values from the 12 tests on the low-plasticity Ontario fine-grained soils (Vardanega and Bolton 2013). Also note that γ_{ref} is strongly correlated with I_p (Vardanega and Bolton 2013).
[b]SD of α for the whole database ~0.12 (Vardanega and Bolton 2013).

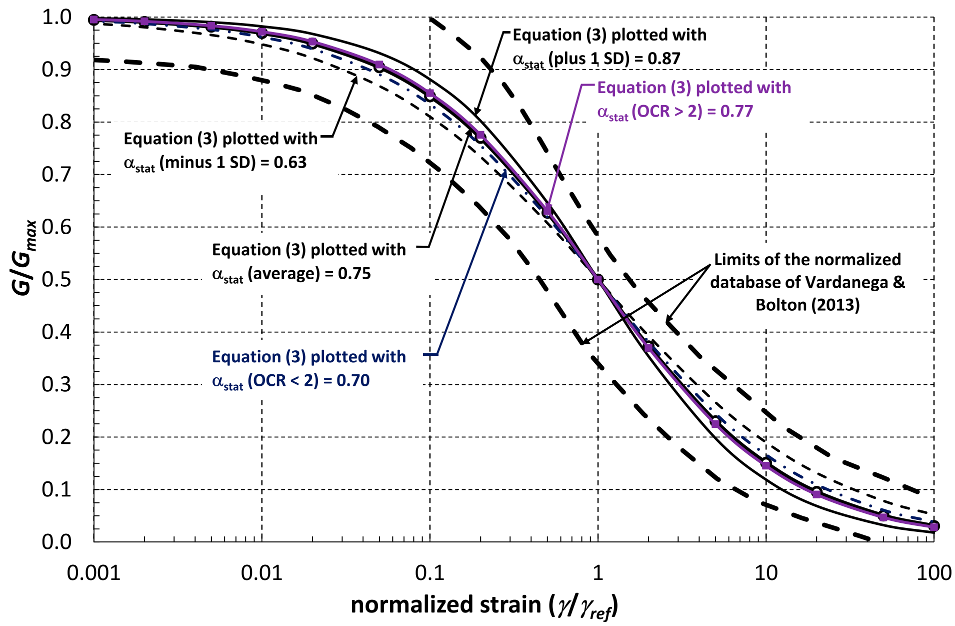

Fig. 4. Variation of the curvature parameter (α) within the database (static adjustment applied)

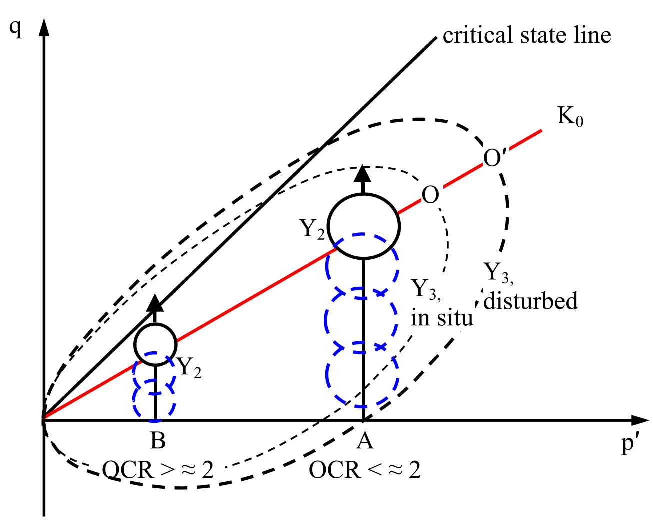

Fig. 5. Kinematic yielding representation

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