

The significance of pore water
pressures on press-in piles

by

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I hereby declare that, except where specifically indicated, the work
submitted herein is my own original work.

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Technical Abstract

Press-in technologies are a comparatively new displacement piling technique, which can offer significance advantages over conventional piling technologies. When compared with equivalently-sized replacement piles, press-in piles boasting benefits of superior stiffness and capacity. Furthermore, when faced with sensitive construction conditions, press-in piling techniques offer improved noise and vibration conditions when compared with driven piles. They are thus becoming a popular piling technique.

The generation of pore water pressures around a press-in pile is a typical for jacked installations into cohesive soils. In the short-term, excess pore water pressures help to reduce the resistance to penetration. Over the long-term, the dissipation of these excess pore water pressures and the regain of effective stress serves to improve the capacity of the pile.

Field tests were performed in 2011 in Kochi, Japan, on a site of normally consolidated, alluvial soils. Two closed-ended instrumented piles were used to analyse the generation and distribution of excess pore water pressures around a third closed-ended installation pile. The two “monitoring” were positioned at various distances, up to a distance of 11 pile radii from the installation pile. Each pile was fitted pore water pressure sensors, earth pressure sensors and load cells along the shaft and in the pile base. On installation of the third “install” pile, the behaviour of the foundation soil could thus be recorded. Various press-in installation techniques were employed, including axial jacking, surging, gyro-pressing and gyro-surfing.

A theoretical model of a pile installation was also considered, assuming the behaviour of the soil around the base of the installation was similar to that around an undrained spherical cavity expansion in non-linear elastic/perfectly plastic soil. The model was calibrated with the results from CPT site investigations, in order to make comparisons with field data.

The generation of excess pore water pressure was most clearly observed during fast installations, where undrained conditions were achieved. In slow installations, the presence of sandy layers presented a large resistance to penetration, and resulted in the rapid dissipation of pore water.

During the installation, the magnitude of excess pore water pressures indicated the presence of a “pressure bulb” — a region of high stress — concentrated around the base of the installation pile. In all tests conditions, the pressure bulb was shown by the monitoring sensors, which recorded significant increases in pore water pressure on the passing of the pile base. After further penetration, the pore water pressures relaxed back to residual values. These trends were

observed by sensors positioned both in the near-field and the far-field

The general trend in results recorded by the remote sensors revealed a decrease in the magnitude of peak pore water pressure with increasing distance from the install pile. The variation in residual values was not so distinct — residual values of excess pore water pressure were approximately constant across all radii. Empirical results were also used to demonstrate a correlation between the magnitude of peak pore water pressures and the residual pore water pressures.

The theoretical analysis was shown to be suitable for the application of predicting the generation of excess pore water pressure at Nunonshida. The spherical cavity expansion analysis, was found to be a good estimate for the lower bound of peak pore water pressures recorded by the monitoring piles.

The combination of experimental and theoretical research has offered a greater insight into the distribution of pore water pressures around an installation pile. It also offers the conclusion that the generation of pore water pressures can, in part, for soils that confirm with the idealised model for non-linear elastic/perfectly plastic soil, be predicted by spherical cavity expansion analyses. The corresponding behaviour of a soil during and after the installation can thus be inferred. This will offer benefits to contractors and designers alike, who may want to use excess pore water pressures for reduced penetration resistances during installation, and enhanced long-term stiffness and capacity.

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Nomenclature

Abbreviations

CPT	Cone Penetration Test
ESP	Effective stress path
mBGL	meters below ground level
PPT	Pile Penetration Test
SPT	Soil Penetration Test
TSP	Total stress path

Nomenclature

c_u	undrained shear strength
c_v	coefficient of permeability
D	diameter of the pile
G	shear moduls, the suffix y denoting the shear modulus at elastic strain γ_y
p	deviatoric stress
p'	effective stress
p_0	the <i>in situ</i> horizontal stress
p_y	pressure at cavity wall at onset of fully plastic behaviour
q	deviator stress
Q_b	base resistance, $Q_{b,peak}$ being the peak base resistance encountered during pile installation
r	radius from centre of cavity expansion. With the suffix 0 denoting the initial cavity radius; with the suffix c denoting the radius of the cavity wall; with the suffix y denoting the elastic-plastic interface
u	pore water pressure (PWP), u_0 being ambient pore water pressure

Δu	excess pore water pressure, Δu_{peak} being the peak excess pore water pressure recorded during an installation, and $\Delta u_{residual}$ being the undrained residual pore water pressure, at the end of an installation
v	resultant velocity of installation pile, given by the sum of the square of v_z (the axial velocity) and v_θ (the tangential velocity from pile rotation)
z	pile toe depth below ground level. With suffix i denotes the depth of a sensor below ground level; with the suffix <i>rel</i> denotes the “relative depth,” the vertical distance between the pile toe and a sensor
α	stiffness constant
β	exponent of elasticity, where 1 is linear elastic
γ	shear strain
γ_y	shear strain at onset of fully plastic behaviour
γ, γ'	unit weight of soil of with suffix w denotes unit weight of water, bouyant unit weight of soil
$\gamma_{M=2}$	mobilisation strain
σ	stress, usually carrying the suffix r or θ for radial and circumferential stress. The suffix y is sometimes used in compound, and denotes the value of stress at yield
τ	shear stress, suffix <i>mob</i> denoting the mobilised shear strain

1 Introduction

When it comes to installing a pile foundation, design and contracting engineers are faced with a number of challenges. Designers would typically concern themselves with issues of pile performance, such as long-term capacity and structural efficiency. For contracting engineers, it was the speed and ease of installation that is of paramount importance. For both parties, an understanding of the pore water pressure distribution around a installation pile can be of great value.

1.1 Motivation

Previous research (Randolph, 2004) has shown that in undrained soils, large positive excess pore water pressures generated during installation help to decrease the resistances against pile penetration. For contractors, this reduces the need for heavy and costly temporary works.

For the design engineer, the dissipation of excess pore water pressures have been shown to be advantageous for the long-term performance of piles. Recent studies (e.g. Jackson, 2008) have demonstrated a process known as “set-up,” where the capacity of the pile increases over time, as pore water pressures dissipate and effective stresses are regained.

However, it should be noted that the ability of a soil to generate excess pore water pressures during an installation, is closely related to the soil characteristics, and the installation technique. “Press-in piling” is a relatively new installation technique, and thus, few studies have considered its ability for generating excess pore water pressures. It thus seems an appropriate focus for this report to consider the significance of pore water pressures on press-in piles.

1.2 Introduction to press-in piling

“Press-in” piling is a comparatively new technology, which has seen much of its development in Japan. A typically installation will involve pressing a solid- or hollow-sectioned pile into the ground by means of a hydraulic jacking system. Machinery, like the Giken piler shown in Figure 1, gains the reaction force for pile installation by using the capacity of previously installed piles — referred to as the “ground reaction” principle. This reduces the need for temporary works, reducing costs and speeding up installation.

Whilst conventional techniques, like pile driving, have been criticised for causing human annoyance, disturbance to sensitive equipment and structural damage to buildings (Boyle, 1990; Crockett, 1980; Head and Jardine, 1992), press-in piling has been recognised for its environmental sensitivity and flexibility. Consequently, examples of its use can be seen world-wide,

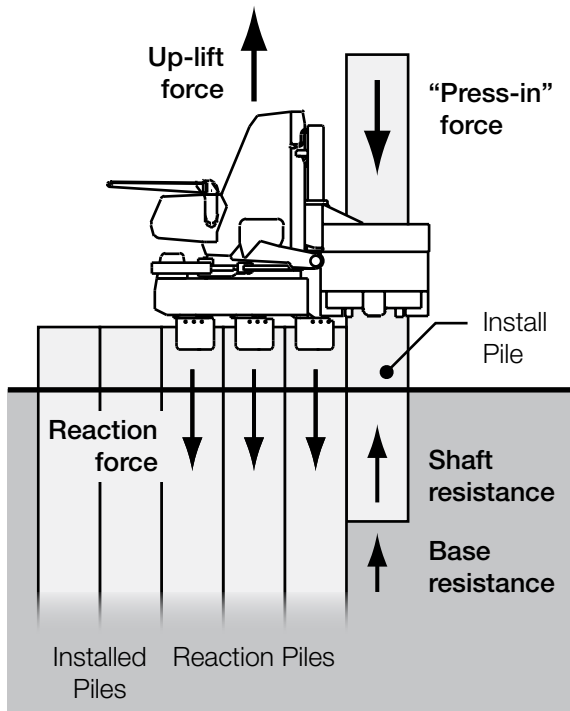


Figure 1: A “press-in” piling machine makes use of the “ground reaction” principle, reducing the need for temporary works



Figure 2: Suitable for use on marginal sites: a pile-jacking machine sitting atop previously installed piles in the docks in Canary Wharf

with demonstrations in London’s Canary Wharf (as shown in Figure 2) and adjacent to railway lines in Perth, Australia.

1.3 Aims

The purpose of this report is to investigate the significance of pore water pressures on press-in piles. It is intended that particular attention shall focus on the generation of excess pore water pressure on the installation of closed-ended tubular piles. More specifically, this report intends to achieve the following:

- Prepare a simple analytical model of pore water pressure behaviour around a closed-ended tubular pressed-in pile in cohesive soils, by considering relevant theory, and previous research.
- Investigate, through practical field tests, the actual behaviour of soil and pore water pressures around a closed-ended pressed-in pile in cohesive soils.

- Perform a comparison between the analytical model and the field results to determine the model's applicability to real installations.
- Use the combined results of the analytical model, and the field test data, to justify the behaviour of a pile during press-in. Specifically, it is hoped that the analysis will serve to predict the behaviour of future installations of press-in piles, both in the short-term and the long-term, with regards to stiffness and capacity.
- Make recommendations for further research, which could direct future work into productive areas.

2 Background theory

To consider the distribution of pore water pressures around a pile, it is first necessary to understand the mechanisms that occur during installation. This section is intended to introduce these mechanisms and highlight their significance with regards to the generation of pore water pressures.

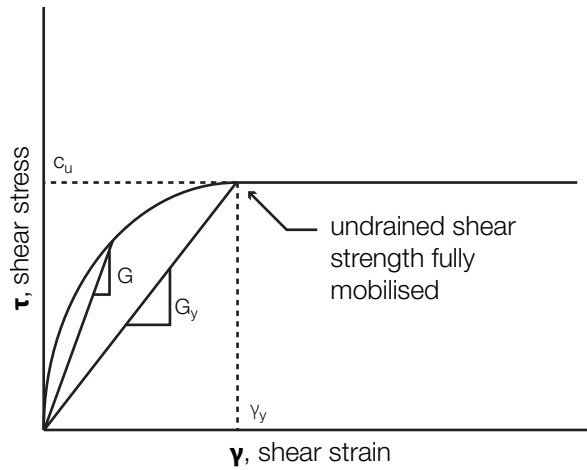


Figure 3: Shear stress:shear strain response for ideal non-linear elastic/perfectly plastic soil

2.1 Non-linear elastic/perfectly plastic soil

Throughout this paper, it has been assumed that the behaviour of soil is non-linear elastic/perfectly plastic. This idealisation is intended to be representative of a cohesive soil — a fine-grained soil whose particles stick together when wet or dry — whose idealised behaviour is presented in the plot of shear stress versus shear strain, in Figure 3.

For small values of shear strain, the elastic response is non-linear, with the soil stiffness, G , decreasing with increasing strain. A fully plastic response is achieved when the value of strain reaches a critical value, γ_y .

The non-linear stress-strain behaviour exhibited by cohesive soils has been found to fit a power law function (Bolton et al., 1993; Gunn, 1992). The constitutive equation describing the variation of shear stress with strain typically takes the form

$$\tau = \alpha \gamma^\beta \quad (1)$$

where α and β are soil properties that need to be found from soil test data.

Recently, Vardanega et al. (2012) published a review of 115 tests, as part of a research programme investigating the stress-strain behaviour of 19 clays and silts. Using the principle of “mobilisation strength”, Vardanega et al. proposed a modified version of the constitutive equation (Equation 2), calibrated against the database of results.

$$\tau_{mob}/c_u = 0.5 \left(\frac{\gamma}{\gamma_{M=2}} \right)^\beta \quad (2)$$

This modified equation shall form the basis of subsequent analysis.

2.2 The behaviour of soil around an installation pile

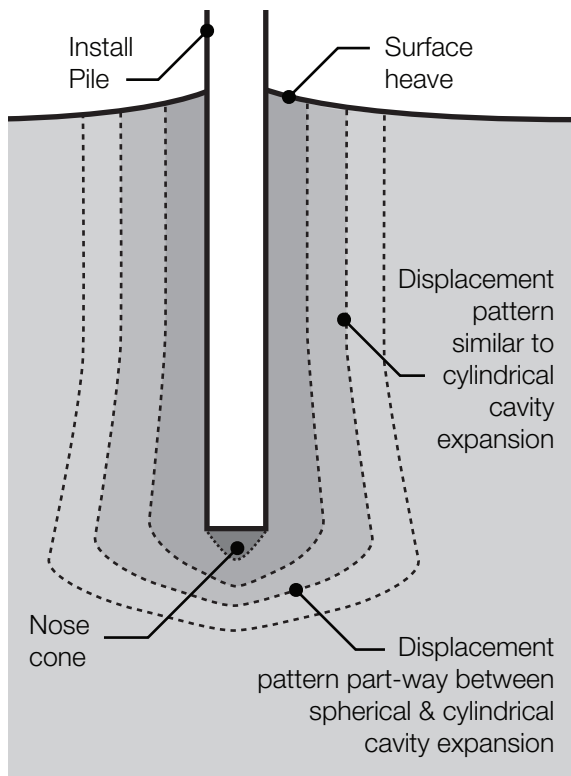


Figure 4: Indicative displacements of soil during a typical pile installation

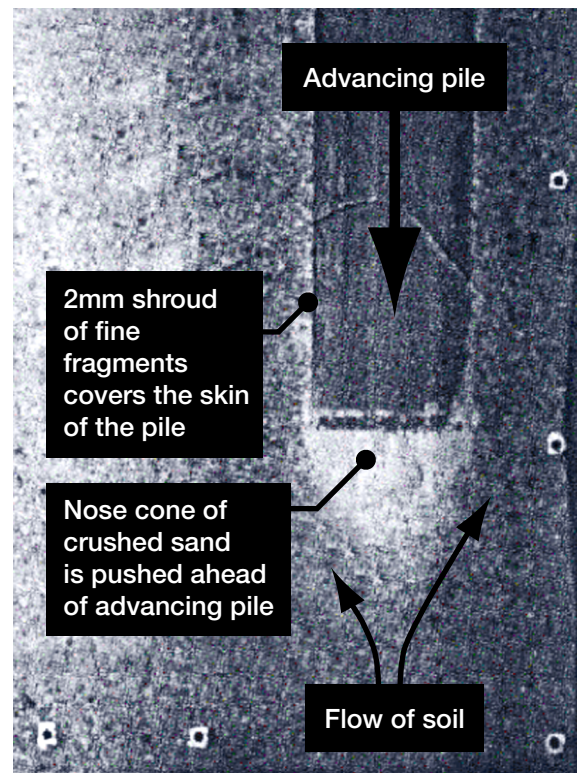


Figure 5: The “nose cone” - crushed soil particles beneath the pile tip. (Photo modified from Bolton and Cheng, 2002)

The behaviour of the soil around a pile can be considered to fall into one of four regimes. Two of these regimes – the “nose cone,” and the “pressure bulb” – are observed around the pile base. The others can be observed at the surface (surface heave), and around the pile shaft (the radial displacement zone). An explanation of each regime is given on the following page. Each regime is also indicatively illustrated in Figure 4.

The nose cone

As a solid pile is initially pressed into the surface of a foundation soil, there is a region around the pile tip where extensive deformation of the soil occurs. Immediately beneath the pile tip, high stresses, resulting from the applied load, serve to crush the underlying soil particles into small pieces. During a typical installation, this region of crushed particles may develop into “nose cone” (Bolton and Cheng, 2002; White and Bolton, 2004; Deeks, 2008), as illustrated in Figure 5.

The “pressure bulb”

Further away from the pile tip, the foundation soil simply flows around the “nose cone” to make way for the advancing pile. The displacement of soil around the pile tip has been shown to be part-way between that associated with the expansion of a spherical cavity and that of a cylindrical cavity (Clark and Meyerhof, 1972; Roy et al., 1975). The corresponding stresses in this region are colloquially referred to as a “pressure bulb.”

Surface heave

Near the surface, the confining pressure from overlaying soil is relatively small. Consequently, as a solid pile is pressed into the ground, the displaced soil (equal in volume to that of the installed pile) has a tendency to “heave” (e.g. Zeevaert, 1950; Hagerty and Peck, 1971; Cole, 1972; Gue, 1984).

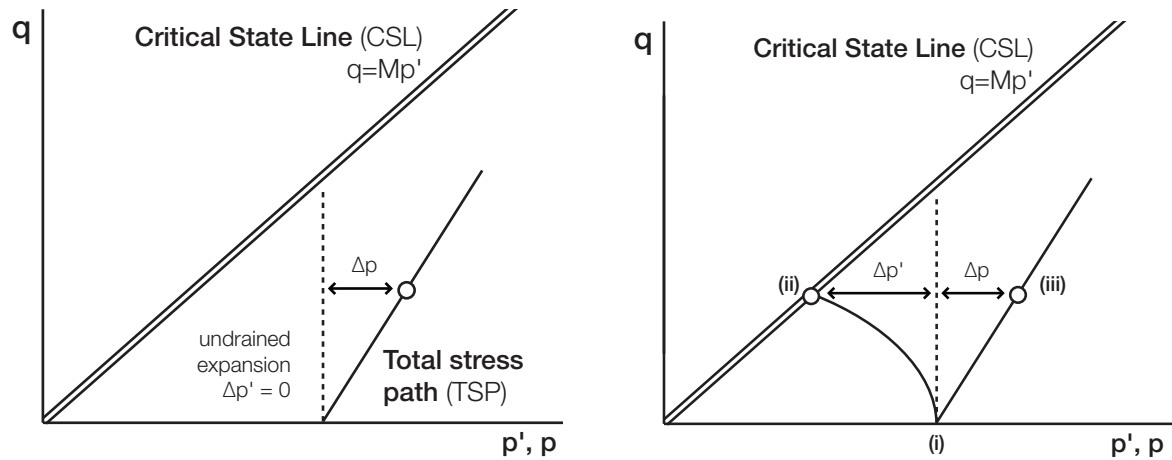
Radial displacement zone

At mid-depths, around the shaft of the pile, the soil is predominantly displaced in a radial direction (Randolph et al., 1979). As a consequence, for deep foundations, the installation pile is often modelled as the expansion of a cylindrical cavity (Soderberg, 1962; Butterfield and Banerjee, 1970).

2.3 The generation of excess pore water pressures, Δu

The installation of a pile, and the displacement mechanisms described above, causes extensive shearing of the foundation soil. The ability for this shearing to generate excess pore water pressures is dependent on the soil properties, described in Section 2.1.

It is well known that shearing an isotropic, linear elastic soil does not generate excess pore water pressure. However, Bolton and Whittle (1999) showed that this was not the case for non-linear elastic material, concluding that changes in the mean normal stress (Δp) during undrained shearing causes a direct increase in excess pore water pressure (Δu). A illustration showing the stress path for a non-linear elastic soil in $q - p - p'$ space has been shown in Figure 6(a).



(a) Pore pressures in an undrained non-linear elastic soil, after Bolton and Whittle (1999)

(b) Pore water in an undrained Cam-clay soil, after Schofield and Wroth (1968)

Figure 6: The generation of pore water pressures in non-linear elastic/perfectly plastic soil

If the soil was assumed to behave in the manner described by critical state mechanics (Schofield and Wroth, 1968), then once the pile reaches plastic yield, yet greater excess pore water pressures could be generated, as illustrated by Figure 6(b). Assuming the soil is initially normally consolidated and pore water pressures equal zero, the critical state model prescribes that the sample, when compressed and sheared by an advancing pile, will follow the effective stress path (i) – (ii). The total stress vector, however, follows the path indicatively shown by (i) – (iii). Whilst the exact trajectory of the total stress path will depend on the assumptions of the test conditions, it is clear that the horizontal distance between the total and effective stress paths ($p - p'$) will account for the generation of excess pore water pressures.

2.4 The importance of excess pore water pressures, Δu

2.4.1 Short term effects of Δu : drained and undrained installations

Corroborating the theory that describes the generation of excess pore water pressures have been numerous studies of real pile installations into cohesive soils (e.g. Seed and Reese, 1957; Soares and Dias, 1989; Lehane and Jardine, 1994). In these tests, it has been observed that large excess pore pressures develop adjacent to the pile surface. The presence of large Δu serves to reduce the local effective stresses in the soil, which, in turn, help to reduce the penetration resistance during installation (e.g. House et al., 2001).

As might be expected, the specific reduction in penetration resistance, is heavily dependent on the drainage conditions. The research of Randolph (2004), shown in Figure 7, confirms this expectation.

It should be noted that in some low permeability soils (e.g. clays or silts), undrained installation behaviour may be exhibited, even at slow installation rates. Typically, the installation of press-in piles is fast enough to make it unlikely that installation is fully drained (White et al., 2010).

It should also be noted that in certain soils, penetration resistance may actually increase in the undrained case when compared with the drained case. The work of Silva et al. (2005), illustrated this behaviour through installations into dilatant, highly over-consolidated clays. The increase in resistance was attributed to the generation of negative pore water pressures around the install pile.

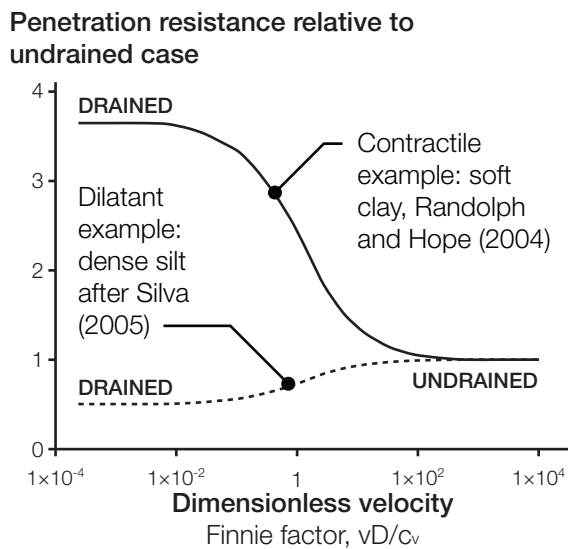


Figure 7: Penetration Resistance (White et al., 2010)

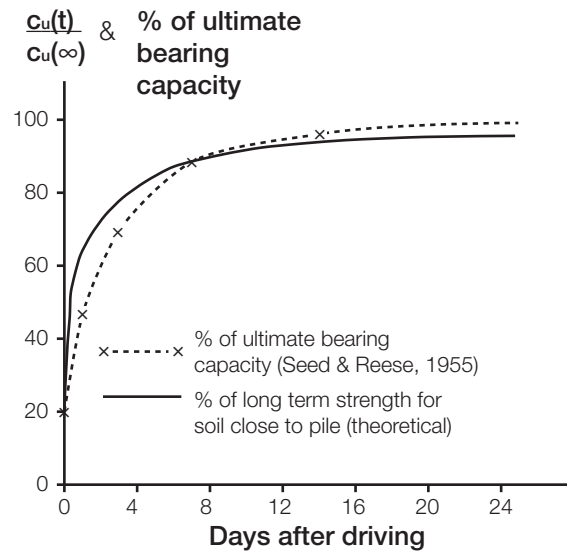


Figure 8: Set up (Randolph et al., 1979)

2.4.2 Long term effects of pore water pressure: “set-up” and “set-down”

In the long term, excess pore water pressures dissipate and effective stresses are regained in a process known as “set-up” (Chow, 1997; Jardine et al., 2006; Jackson, 2007). In tests by Seed and Reese (1957), the final “equalised” values of radial effective stress were approximately five times greater than those measured at the end of pile installation. In the same tests of Seed and Reese, the increase in radial effective stresses resulted in an increase in soil strength - hence the capacity of the pile was improved. The rise in pile capacity, as a function of time is included in Figure 8, above.

Similar to “set-up”, the dissipation of negative pore water pressures has an affect on the radial stresses acting on the pile. Mitchell and Manders-Jones (2004) observed this process (known as “set-down”), recording a drop in pile capacity over time.

2.5 Cavity expansion analyses

To better understand the behaviour of pore water around a pressed-in pile, it is the intention of this report to build a simple analytical model simulating the pile installation. Previous studies have shown that cavity expansion analyses are particularly useful in determining the ultimate end bearing capacity of pile (e.g Vesic, 1972; Randolph et al., 1979, 1994). It is hoped that these analyses will offer a similarly valuable insight into the distribution of pore water pressures.

Cylindrical cavity expansion analyses were first conducted in the field of geotechnics by Gibson and Anderson (1961) and Meyerhof (1951). The concept of cylindrical or spherical cavity expansion is similar to the concept of inflating a balloon. It is first assumed that the “analogous balloon” has been installed into the ground, to the appropriate depth. The balloon has an initial radius, $r_0 = 0$, and causes no disturbance to the soil during installation. The balloon (either cylindrical or spherical in respective cases) is then inflated. For the purpose of comparing these expansions with pile installation, it is assumed that the balloon is inflated until the outer radius of the balloon is the same as that of the pile ($r_c = r_{pile}$).

2.6 The stress distribution around a cavity expansions in non-linear elastic/perfectly plastic soil

For cavity expansions in non-linear elastic/perfectly plastic soil, there is the potential to develop two soil regimes: one elastic, one plastic. In a pile installation, it is assumed that both regimes will develop, as illustrated in (Figure 9).

In a recent paper, Cao et al. (2002) used this analogy to prepare a detailed derivation for the distribution of stresses and pore water pressures for cavity expansions in non-linear elastic/perfectly plastic soil. The analytical model of soil behaviour used in this report directly uses the work of Cao et al., and key equations have been included, for reference, in the Appendix.

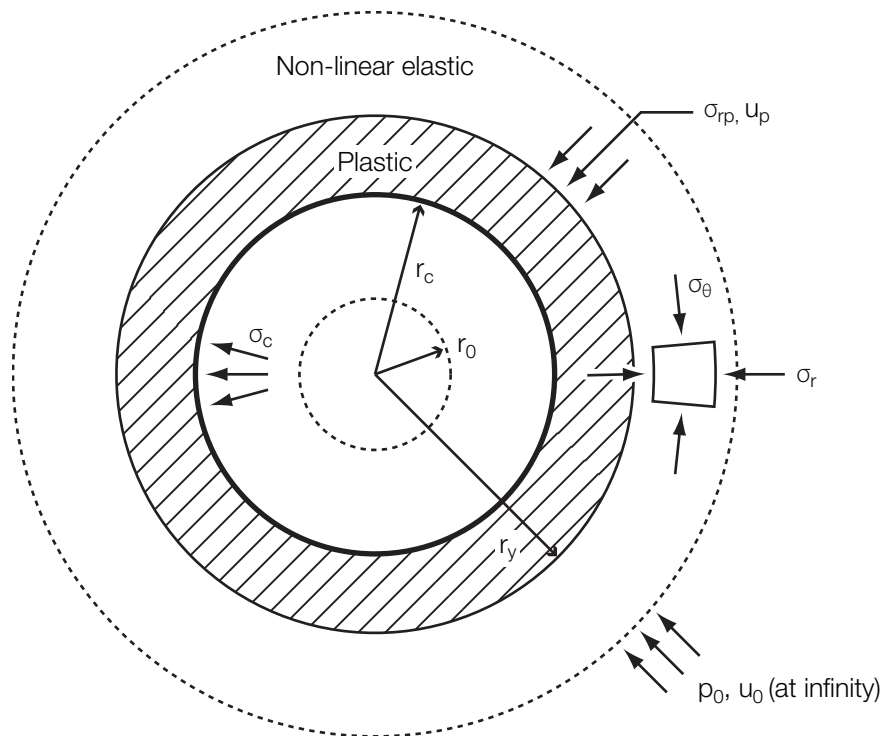


Figure 9: Expansion of a cavity, after Cao et al. (2002)

3 Design of experiment

To assess the affect of distribution and significance of pore water pressures around pile installation, 18 field tests, using three instrumented piles, were conducted at Giken's Nunoshida Test site in Kochi. This section presents the design of the test plan, the soil properties at Nunoshida, and the methodology that was undertaken.

3.1 Layout of tests

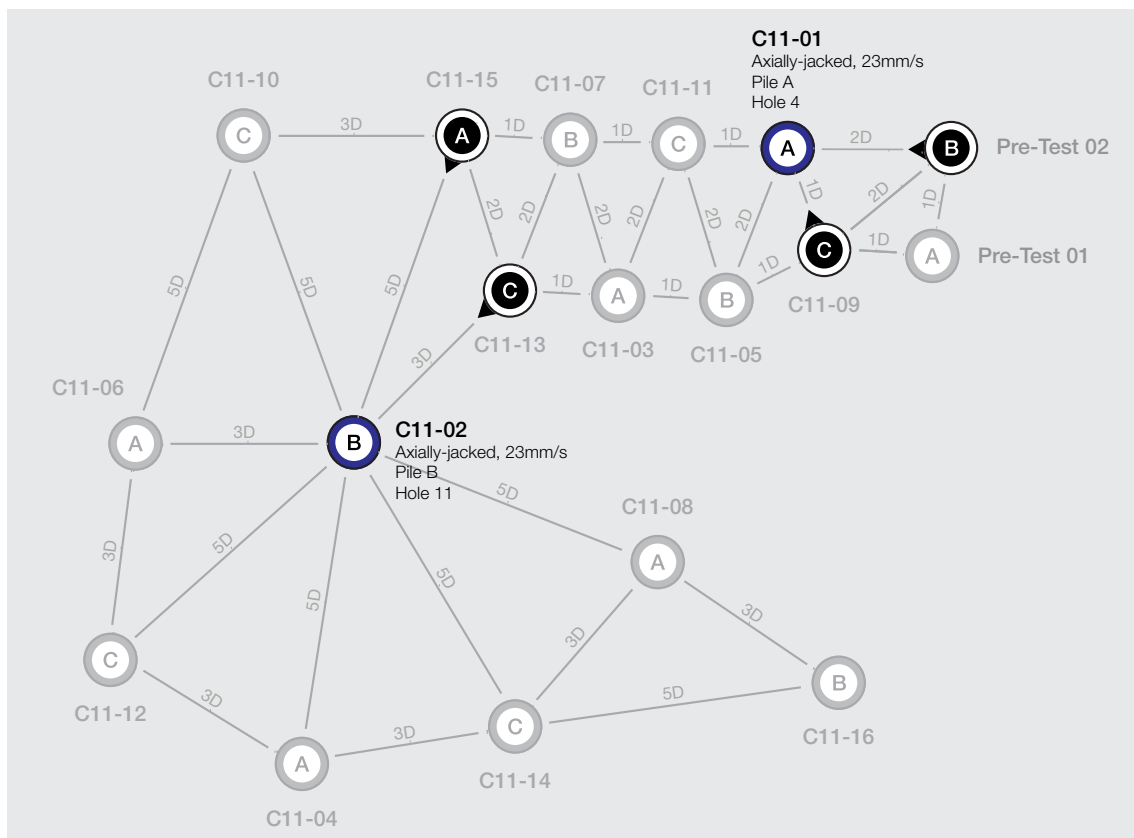


Figure 10: The layout of tests

The test layout, illustrated in Figure 10, was devised in order to record the variation of pore water pressures with radial distance. Each installation is coupled with another — e.g. C11-01 and C11-02 — insofar as the installation conditions are the same in the two tests (see Section 3.6). Across the two installations, “monitoring piles” fitted with sensors record the changes in total pressures, and pore water pressures. These monitoring piles were located at pile-wall-to-wall distances of 1D, 2D, 3D and 5D respectively (where D is 318.5mm, the pile diameter). One pair of installations, and their corresponding monitoring piles has been highlighted in Figure 10.

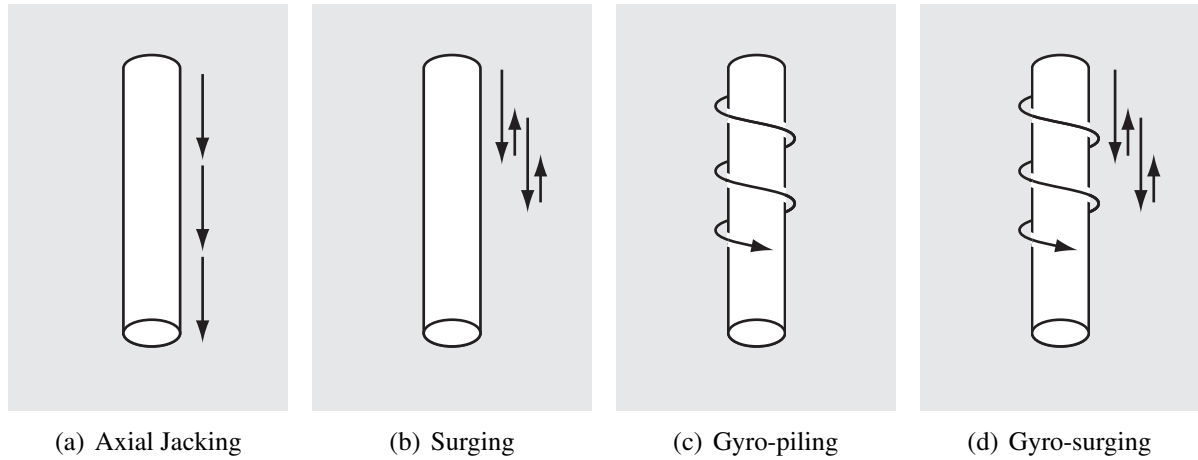


Figure 11: Installation techniques (arrows indicative of pile motion)

3.2 Installation techniques

The test piles were installed in one of four ways: either axially jacked (also known as monotonic jacking), surged, gyro-piled or gyro-surfed. Each one of these installations was driven at a pre-defined speed (see Section 3.6). Illustrations of these different techniques are included in Figures 11(a) – 11(d).

Axial Jacking

In Axial Jacking, the pile is pressed into the ground for one working stroke. The chuck, which clamps the pile, is then released. The chuck re-clamps and pushes the pile downwards for another stroke, until the pile is fully installed to 12mBGL.

Surging

Surging is an installation technique that involves “vertical cycling” of the pile, which is known to ease installation (White and Deeks, 2007). The length of the downwards stroke is longer than the upwards stroke, as defined in Table 1. The net effect of surging is to advance the pile into the ground. The installation surge continued until the pile is fully installed to 12mBGL.

Rotary jacking

Rotary jacking (also known as gyro-piling) is a “screwing” motion — the pile is simultaneously rotated whilst being axially jacked into the ground.

Gyro-surfing

Gyro-surfing is the combination of surging and rotary jacking. The piles were simultaneously “surged” and “screwed” into the ground.

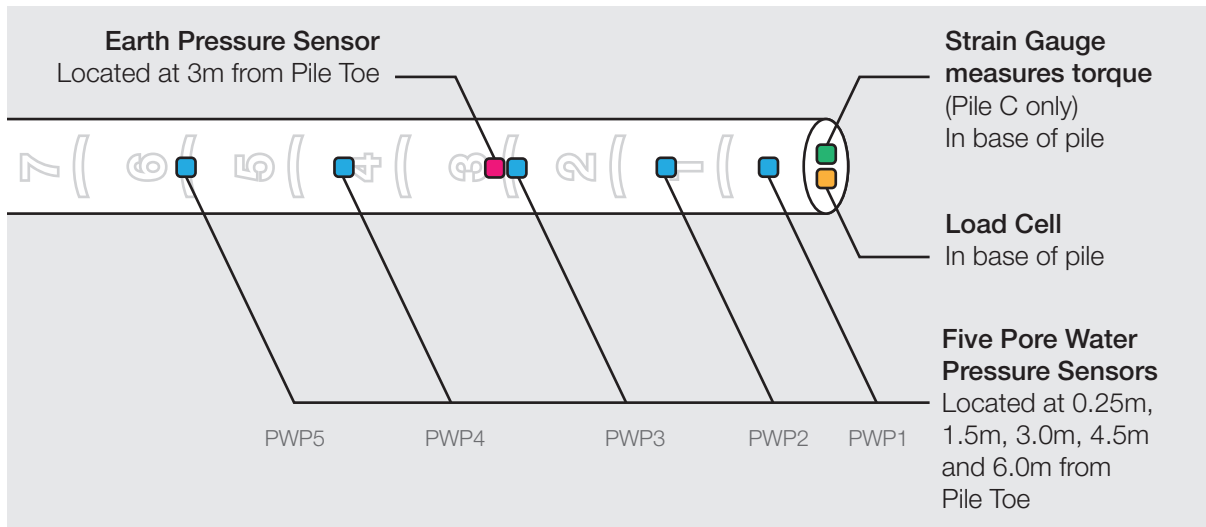


Figure 12: Illustration of the instrumented test pile

3.3 The test apparatus

The test piles

Each of the three instrumented piles (test piles A, B, and C) was closed-ended, 318.5mm in diameter, and fitted with the following sensors:

- 5 pore water pressure sensors, fitted at 0.25m, 1.5m, 3m, 4.5m and 6m from the pile toe
- One earth pressure sensor, fitted at 3m from the pile toe
- One load cell to measure the pressure on the base of the pile

Pile C was fitted with an additional strain gauge, used to measure the base torque during installation. An illustration of the test pile is shown in Figure 12.

The “strokemeter”

To measure the embedment of the pile, a coiled wire extensometer (a “strokemeter”) was used, anchored at ground level, and aligned. Two spring-loaded drums of rope ensured the strokemeter was held vertical during the tests.

The piler

A Giken Gyropiler (Model SP3) was used for all 18 tests. Previous tests at the site (Delano, 2010; Bond, 2011) had shown that the maximum resistance against axial jacking would be of the order $\sim 320\text{kN}$, whilst the maximum resistance against torque would be of the order $\sim 21\text{kN}$. To provide the necessary reaction against the anticipated ground resistance, “reaction weight” was added to the piler, taking the form of a “reaction base” and “reaction piles.”

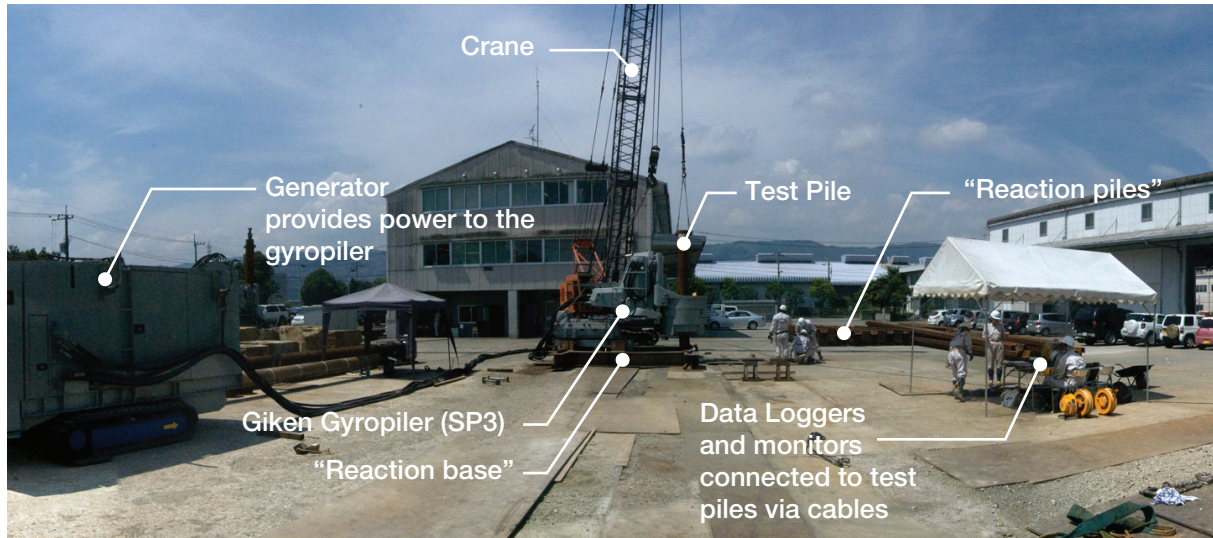


Figure 13: Experimental set-up

3.4 Experimental set-up

Data from each test installation was streamed, in real-time through cable connections, to a data logger (Figure 13). The results were displayed on a monitor positioned just adjacent to the installation site. This made it possible to monitor the results during each test.

A crane, driven by a trained operator, was used on site to assist with manoeuvring the test piles (Figure 13).

The piler, which was controlled by a trained operator, had been pre-programmed with the installation test conditions described in Section 3.6.

3.5 The test procedure

All three of the test piles were used simultaneously during the tests. One of the piles was the “installation pile” and would be pressed into the soil by the piler. The other two piles were static “monitoring piles” — piles previously installed to 12mBGL, with sensors aligned facing the installation pile. With the exception of tests C11-15 and C11-16, each installation involved the five-step procedure, outlined below. Data from the sensors on all three piles data was recorded continuously, with a sampling rate of 2Hz.

Step 1: Calibration

A short calibration procedure took place before the start of each test. The pile would be held in the “chuck” of the piler above the ground surface, then lifted up and down, simulating the conditions of installation. During calibration, the pile never penetrated the ground surface — as such

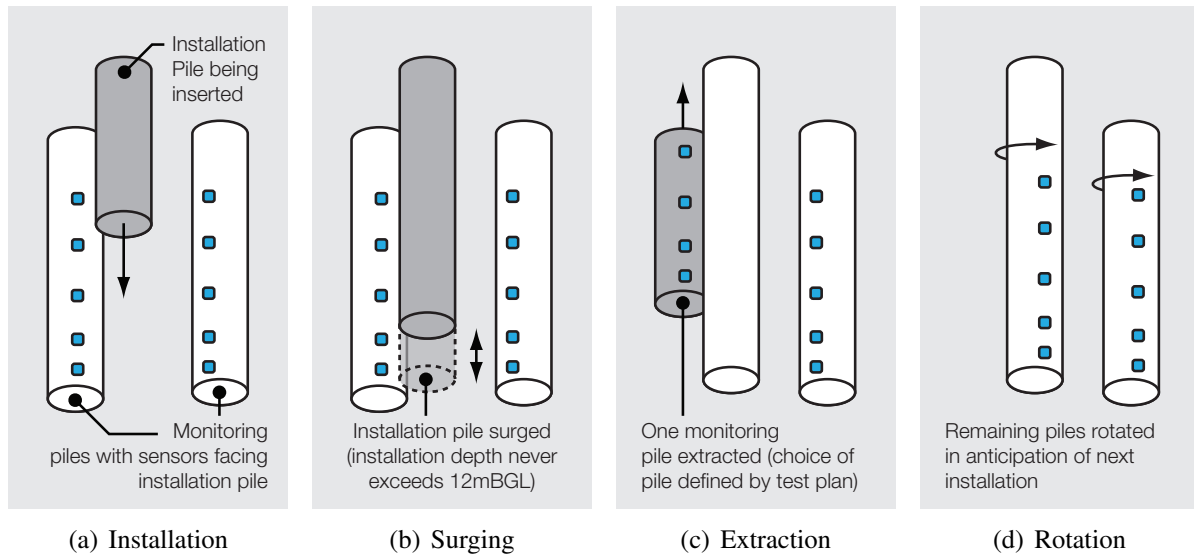


Figure 14: The test procedure

measurements of force were independent of ground resistance, making it possible to identify the force required to lift the pile and the chuck.

Step 2: Installation

The installation pile is jacked to a depth of 12mBGL using the installation procedures described in Section 3.6. The monitoring piles remain static, recording the changes in stress and pore water pressure resulting from the installation (Figure 14(a)).

Step 3: Surging at 12mBGL

The installation pile is then “surged” (Figure 14(b)). The surging process took place at constant depth (i.e. the pile did not penetrate below 12mBGL during this process). The duration of the surging tests were not pre-defined. Instead, the test operators called an end to surging when the on-screen results indicated there were no further changes in values being recorded.

Step 4: Extraction

One of the monitoring piles is then removed (Figure 14(c)). The pile that is removed is prescribed by the test plan described in Section 3.1.

Step 5: Rotation

The two piles remaining in the ground will become the monitoring piles for the following test. They are rotated in anticipation of the next installation (Figure 14(d)).

During tests C11-15 and C11-16, surging took place at approximately 9mBGL (as opposed to 12mBGL). At this depth, the base of the pile would be in sand, and it was hoped that quick surges in this layer might produce negative excess pore water pressures. After surging at this depth, the installation of the pile continued, being pressed-in to 12mBGL. The rest of the test procedure (extraction and rotation) remained unchanged.

It should be noted that only data from the installation process will be presented in this report.

3.6 Installation Conditions

Table 1: Installation conditions

Installation Procedure	Installation No.	Install Rate mm/s	Extract Rate mm/s	Rotation Rate mm/s	Surging stroke mm	Horizontal spacing between installation & monitoring piles	Install pile
Axial jacking	Pre-01	23	-	-	-	-	A
Surging	Pre-02	23	30	-	800-400	-	B
Axial jacking	C11-01	23	-	-	-	1D, 2D	A
	C11-02	23	-	-	-	3D, 5D	B
	C11-03	120	-	-	-	1D, 2D	A
	C11-04	120	-	-	-	3D, 5D	A
Surging	C11-05	23	30	-	800-400	1D, 2D	B
	C11-06	23	30	-	800-400	3D, 5D	A
	C11-07	23	30	-	400-200	1D, 2D	B
	C11-08	23	30	-	400-200	3D, 5D	A
Rotation	C11-09	23	-	110	-	1D, 2D	C
	C11-10	23	-	110	-	3D, 5D	C
	C11-11	23	-	15	-	1D, 2D	C
	C11-12	23	-	15	-	3D, 5D	C
Surging & Rotation	C11-13	23	28	110	800-400	1D, 2D	C
	C11-14	23	28	110	800-400	3D, 5D	C
Surging	C11-15	23	30	-	800-400	1D, 2D	A
	C11-16	23	30	-	800-400	3D, 5D	B

Table 1 lists the installation conditions of all 18 tests. Each test installation was paired with another (e.g. Tests C11-01 and C11-02). The only difference between each test in the pair was the radial separation of the monitoring piles.

It should be noted that the axially-jacked tests were run at two different speeds, the surging tests employed two different stroke lengths, and the gyro-pressed test run at two different rotational speeds.

3.7 The Nunoshida Test Site - soil profile

The soil profile at Nunoshida consists of normally consolidated alluvial deposits — predominantly sands, clays, and sandy clays (Figure 15).

A cone penetration test was conducted at the site, to find the shear strength of the soil strata. The results from this investigation are illustrated in Figure 16. The strength of the clays at 6mBGL and 7.5mBGL have been labelled and will be used in subsequent analysis.

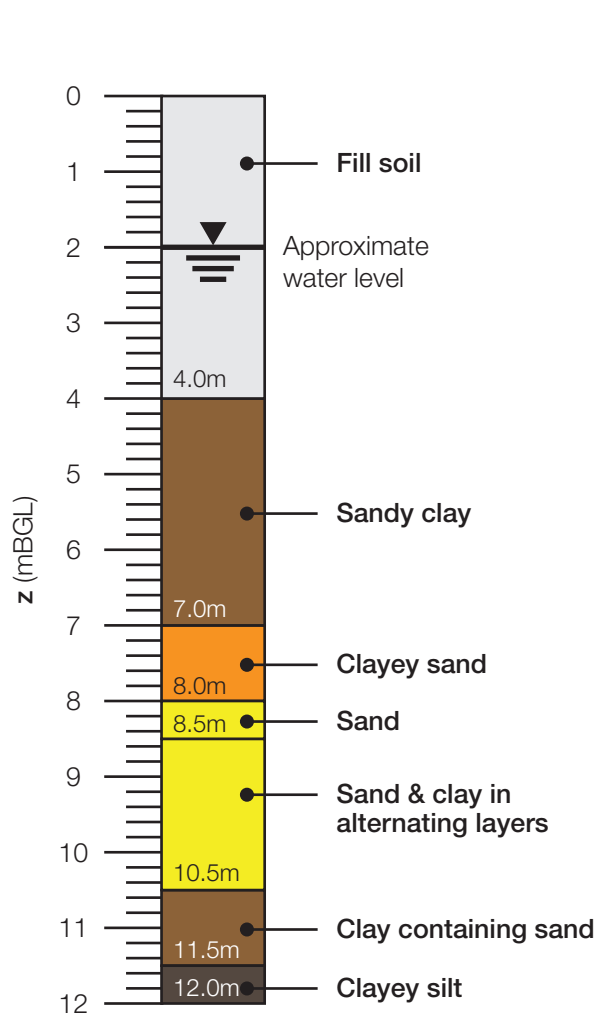


Figure 15: Soil profile at Nunoshida

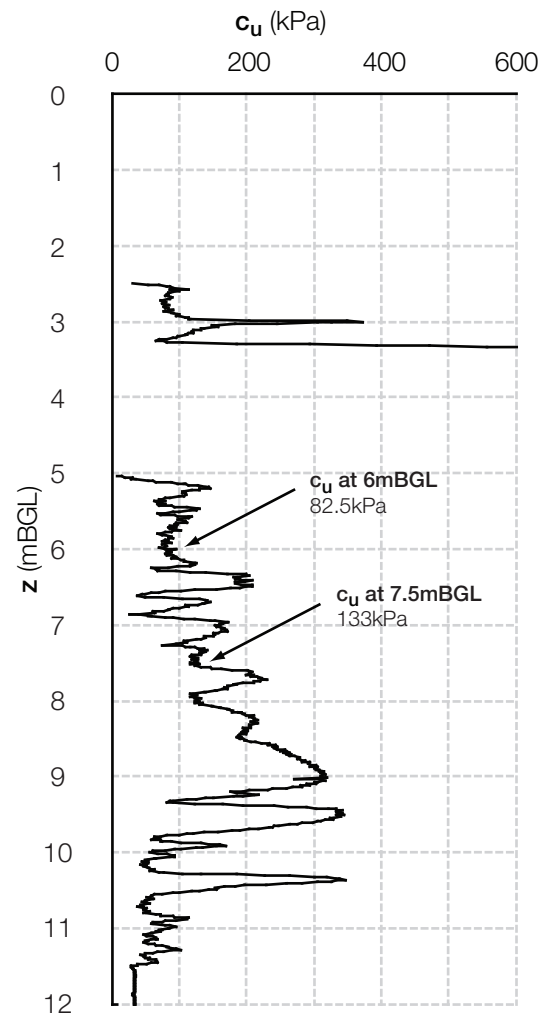


Figure 16: Soil Strength Profile



(a) Pre-augered pilot hole — prepared before testing



(b) Void pipes

Figure 17: Preparations prior to the start of testing

3.7.1 Void pipes

To ensure good results during testing, the top four metres of “made ground” had been removed by using an over-size auger (Figure 17(a)), and cardboard “void pipes” inserted to prevent the holes from collapsing (Figure 17(b)). It was intended that the void pipes would be removed just prior to the start of each installation.

Table 2: Problematic results and corrections

	Source of Error	Piling machine fault. Installation faster than expected	Void pipe not removed. Installation obstructed / surging required	Loose connection to PWP-A1	Loose connection to PWP-C1	Data for Pile C accidentally erased
		Compensate for rate effects	Treat installation data with caution	Discard data for PWP-A1	Discard data for PWP-C1	Data Lost for all sensors on Pile C
Affected Installations	Pre-01		x			
	Pre-02		x			
	C11-01		x			
	C11-02			x		
	C11-03		x	x		
	C11-04	x				
	C11-05					
	C11-06					
	C11-07			x		
	C11-08	x				
	C11-09		x			x
	C11-10			x	x	
	C11-11			x		
	C11-12					
	C11-13			x		
	C11-14	x				
	C11-15			x		
	C11-16					

3.8 Experimental errors

Due to malfunctioning sensors and equipment, there were a few installations that could not be completed according to the pre-defined test criteria. The severity of the errors were typically relatively minor, and thus simply needs to be treated with caution during data analysis. However, in the worst case, a complete data stream was irreversibly lost (test case C11-09) having been accidentally erased on the data logger. A summary of the corrections has been included in Table 2.

4 Base resistances

In this section, results for the base response of the pile during installation have been prepared. Recent research by Ishihara et al. (2011) has shown that the press-in piling technique provides an accurate indication of the soil properties, and the base response can be used to confirm the soil profile at each installation location.

Confirming the soil profile was particularly important given previous field tests at Nunoshida (Bond, 2011) have shown the soil conditions to vary dramatically from that predicted by CPT investigations.

4.1 Initial data processing

The initial results for base resistance have been included in Figure 18. The horizontal lines shown in Figure 18(a) are an undesirable artefact from the installation procedure, relating to readings taken between working strokes, whilst the pile was static. These data points are of no use in determining the soil profile at Nunoshida, so, for clarity, these horizontal have been removed. The results from data processing are shown in Figure 18(b).

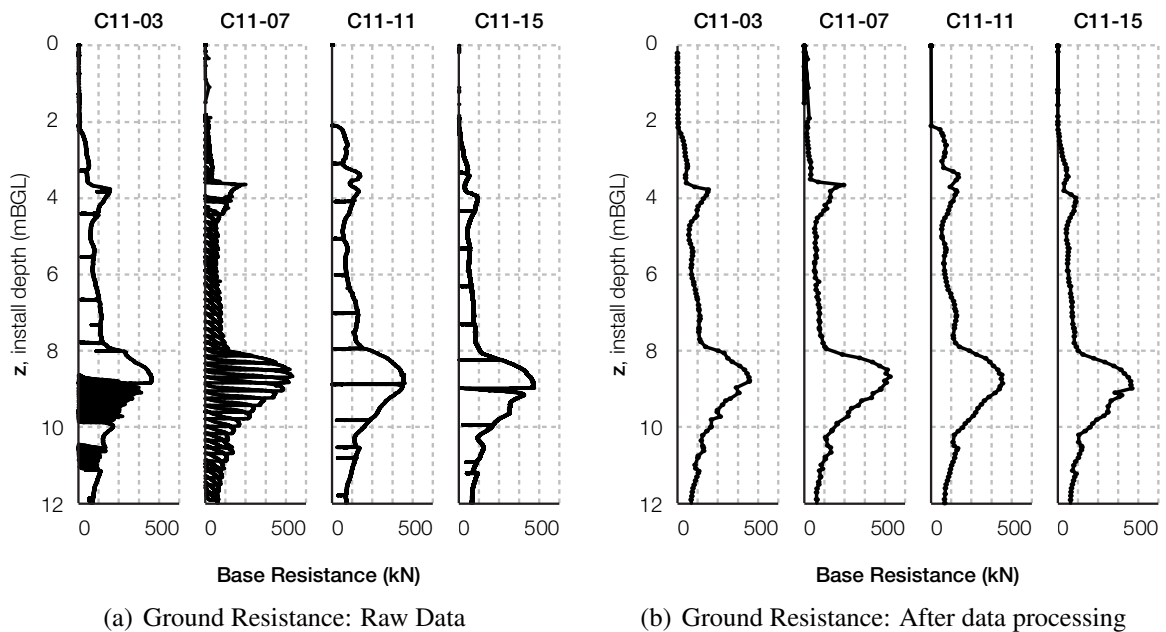


Figure 18: Base Resistances

4.2 Results

4.2.1 Base response (Q_b): comparison across all 16 test installations

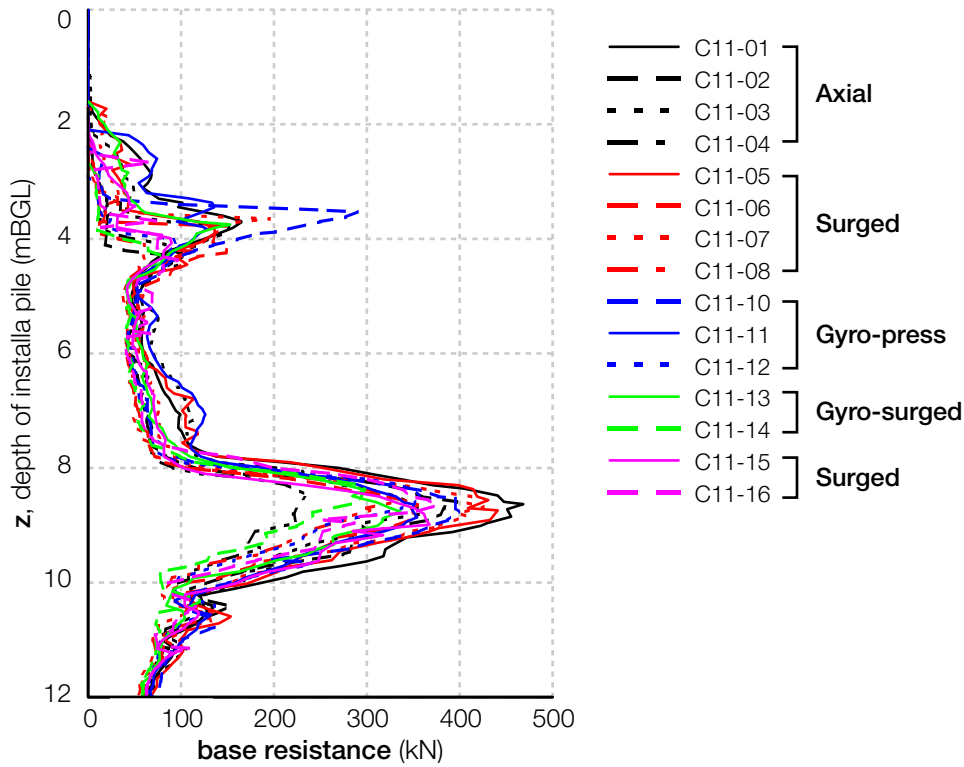


Figure 19: A consistent base response across all 16 tests confirms a consistent soil profile across the site

Figure 19 shows the base resistance (Q_b) plotted against z , as recorded by the load cell in the base of each installation pile. As can be seen, a consistent base response was experienced by each pile across all 16 tests. Peak base resistances ($Q_{b,peak}$) were consistently recorded at a depths of ~ 8.5 mBGL.

Both the largest and smallest values of $Q_{b,peak}$ were recorded during axially jacked installations. The maximum and minimum values were 470kN (C11-01, $z = 8.6$ m) and 230kN (C11-04, $z = 8.5$ m) respectively.

The early peak recorded at a depth of ~ 3.5 mBGL during installation C11-10 (shown as a dotted-blue line) was due to the presence of a stiff layer of “made ground” at the base of the pre-augered pilot hole. In all other tests cases, the made ground had been completely removed. Consequently, this value should be ignored.

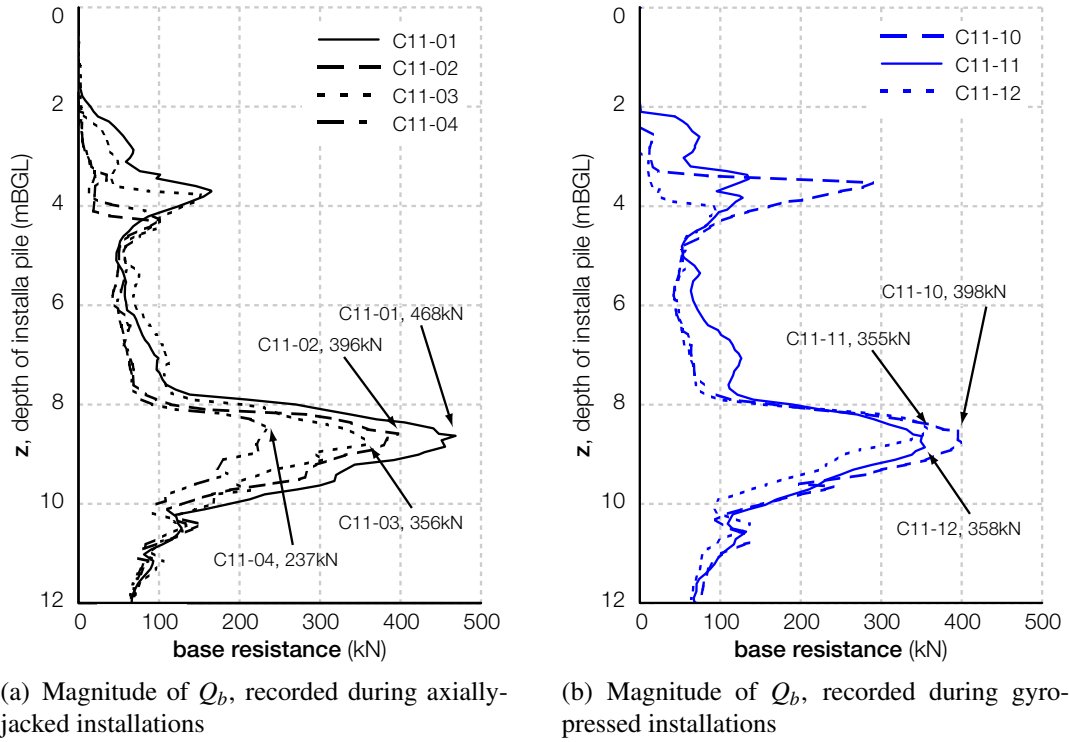


Figure 20: Plots of base resistance for different installation techniques. High resistance encountered by “closely”-grouped piles — a consequence of interference effects

On closer inspection of identical test cases, it was observed that for closely-grouped installations, the base resistance encountered by the install pile is greater than that in the loosely-grouped installation. This is more clearly illustrated in Figure 20(a), where the results from the axial tests have been plotted alone. C11-01 and C11-02 both employ the same test conditions, but C11-01 (closely-grouped) records a value $Q_{b,peak}$ approximately 18% larger than that in test C11-02 (loosely-grouped).

This observation was consistent across all tests except the rotary jacked tests (Figure 20(b)), in which the loosely-grouped installation, C11-12, experienced a marginally larger base resistance ($Q_b = 358\text{kN}$) when compared with the closely-grouped installation C11-11 ($Q_b = 355\text{kN}$).

4.3 Interpretation

From predictions based on CPT site investigations, it was believed that the peak base resistance at Nunoshida would be $\sim 320\text{kN}$. From the field data, the magnitude of $Q_{b,peak}$ was not too dissimilar, averaging at a value of $\sim 360\text{kN}$ (sample size, $n = 15$). The maximum peak base resistance, however, was much larger, at $\sim 470\text{kN}$ (recorded during installation C11-01). This

unexpectedly high base resistance, encountered at depths of between 8mBGL and 10mBGL, posed particular problems during test installations C11-01, C11-03 and C11-09. In order to complete the installation in all three cases, as it was necessary to “surge” at this level.

From inspection of the results, the high base resistance may be a consequence of variable soil strength. If between depths of 8mBGL and 10mBGL there were slightly stiffer soils, the load cells would record a correspondingly high value of base resistance, as compared with installations over the rest of the site. An alternative hypothesis attributes the higher base resistance to “interference effects”. The close “grouping” of piles may decrease the “driveability” of piles, as described in Section 4.3.1, below.

4.3.1 Interference effects

Figures 20(a) – 20(b) illustrate that, across the majority of the tests, the base resistance was greater in the cases of closely-grouped piles when compared with the cases of loosely-grouped piles. Interference effects (also known as “enclosure effects”, Yetginer et al., 2006) are known to decrease the “driveability” of piles (e.g. Tomlinson, 2001) as the presence of a neighbouring pile will impinge on the displacement mechanism around the installation pile.

Interference effects are particularly interesting because they have profound practical implications. Giken’s press-in piling technology is particularly suited to installing piles in close groups. It is likely that the interference mechanism, which generates large base resistances, will also generate large excess pore water pressures. This could have either positive benefits for the long-term stiffness and capacity of permanent foundations, or deleterious effects if it was necessary to extract piles at a later stage.

The work of Yetginer et al. (2006) offers a interesting insight into the benefits of group piling, but further work would be recommended to explore the interference mechanism of pore water pressures, which falls outside the scope of this project.

4.4 Summary

In field tests, the variability of soil can have a significant impact on observations and results. As such, understanding the types of soils around each installation, and their corresponding properties, is of primary importance before further analysis can be completed.

A consistent base response was recorded in all 16 tests – comparing favourably with previous research at Nunoshida (Delano, 2010; Bond, 2011) and with theoretical predictions based on CPT site investigations. It can be concluded, with reasonable confidence, that the soil profile and corresponding soil properties, illustrated in Figures 15 and 16, are an accurate representation of the those encountered by each installation.

The significance of pore water pressures on press-in piles

Part of the variability in the magnitude of peak base resistance is attributed to interference effects. A complete analysis of interference effects is beyond the scope of this report, and would be recommended for further research.

5 A comparison of base resistance and excess pore water pressure (Δu)

With a confident understanding of the soil profile, attention is now turned to the generation of excess pore water pressures during installation.

In this section, the following selection of tests are considered:

Slow Installations

- C11-02, Axially Jacked, $v_z = 23\text{mm/s}$
- C11-06, Surged, $v_z = 23\text{mm/s}$
- C11-08, Surged, $v_z = 23\text{mm/s}$
- C11-12, Gyro-pressed, $v_z = 23\text{mm/s}$, $v_\theta = 15\text{mm/s}$

Fast Installations

- C11-04, Axially Jacked, $v_z = 120\text{mm/s}$
- C11-10, Gyro-pressed, $v_z = 23\text{mm/s}$, $v_\theta = 110\text{mm/s}$
- C11-14, Gyro-surged, $v_z = 23\text{mm/s}$, $v_\theta = 110\text{mm/s}$

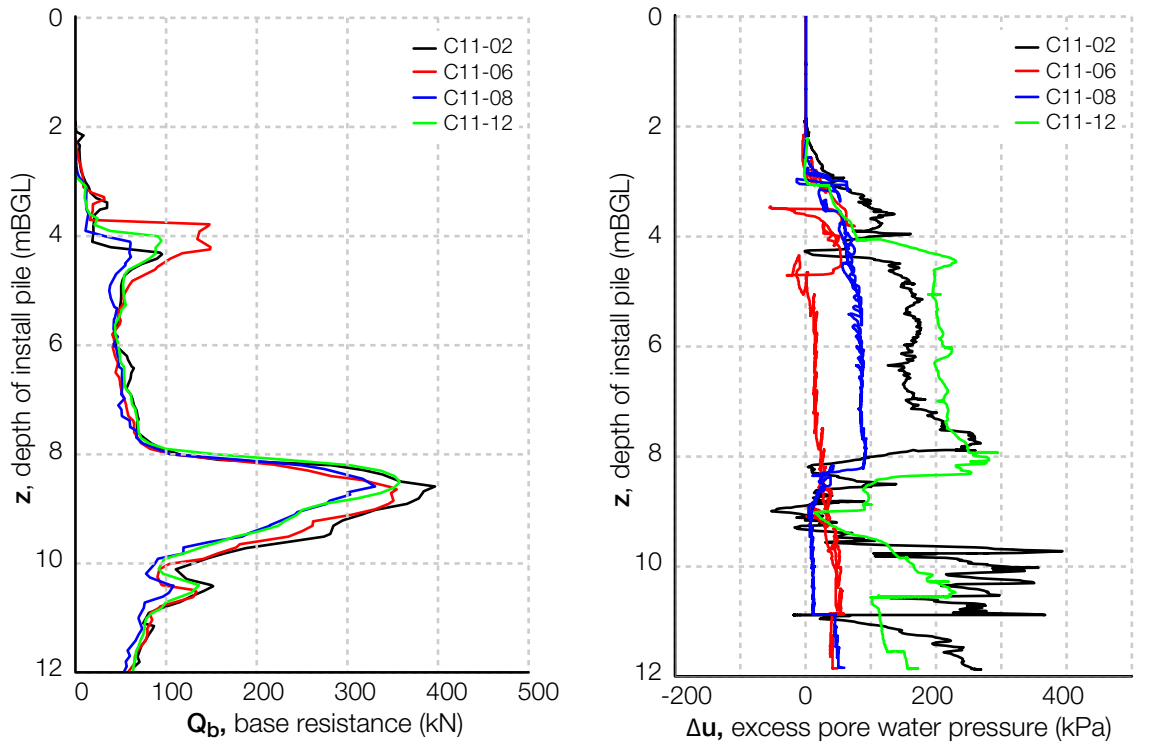
The test cases are split into two groups: slow installations and fast installations. The fast installations, comprise the tests where the resultant installation speed, $v \approx 115\text{mm/s}$. In the slow installations, $v \approx 25\text{mm/s}$.

It should be noted that only the results from the “loosely”-grouped test installations have been considered. As described in Section 4.3.1, it is beyond the scope of this project to compensate for interference effects. In subsequent analysis, the loose grouping of the test cases are assumed to be independent of interference effects.

5.1 Results

5.1.1 Comparison of installation techniques

Slow installations



(a) Q_b , recorded by a load cell in the base of each installation pile (b) Δu , recorded by a sensor 0.25m from the pile toe

Figure 21: Comparison of base resistance, Q_b , and excess pore water pressures, Δu , during slow installation tests

Base response

The results of base resistance, and excess pore water pressures for the slow installations are plotted in Figures 21(a) and 21(b). In each case, the data has been recorded by sensors on the installation pile. The results of Δu relate to the measurements taken by a pore water sensor positioned 0.25m from the pile toe.

In all four slow installations, the base response exhibited almost exactly the same behaviour between depths of ~ 5 mBGL and 8mBGL. Over all depths, the axially-jacked installation (C11-02, black line), typically records the largest magnitude of Q_b , reaching a peak of ~ 395 kN – noticeably greater than those of the other tests. The magnitude of response during rotary-jacking and long-stroke surging installations (green and red lines respectively) are almost exactly the

same. The short-stroke surged installation (C11-08, blue line) encountered the least base resistance at almost all depths.

Excess pore water pressures

The general behaviour in the excess pore water pressures, as shown in Figure 21(b), can be described as follows:

- Between depths of $\sim 5\text{mBGL}$ and 8mBGL , the sensor records a positive value of Δu . In each of the respective test cases, there is typically little variation in the magnitude of Δu at this level.
- At $\sim 8\text{mBGL}$, the excess pore water pressure rapidly drops to almost 0MPa .
- The trends remain at a negligible value until the pile toe reaches a depth of $\sim 10\text{mBGL}$, at which point, the trends in Δu begin to increase.

The results recorded by the pore water pressure sensor in test case C11-06 seems to be inconsistent with the other trends. C11-08 also shows very little change in Δu beyond depths of $\sim 8\text{mBGL}$. It is believed that both of these results are anomalous, and may be due to a small pocket of highly permeable sand being trapped next to the pore water pressure sensor in each case. This would rapidly dissipate any change in pore water pressure, “muting” the values recorded by the sensor.

Fast installations

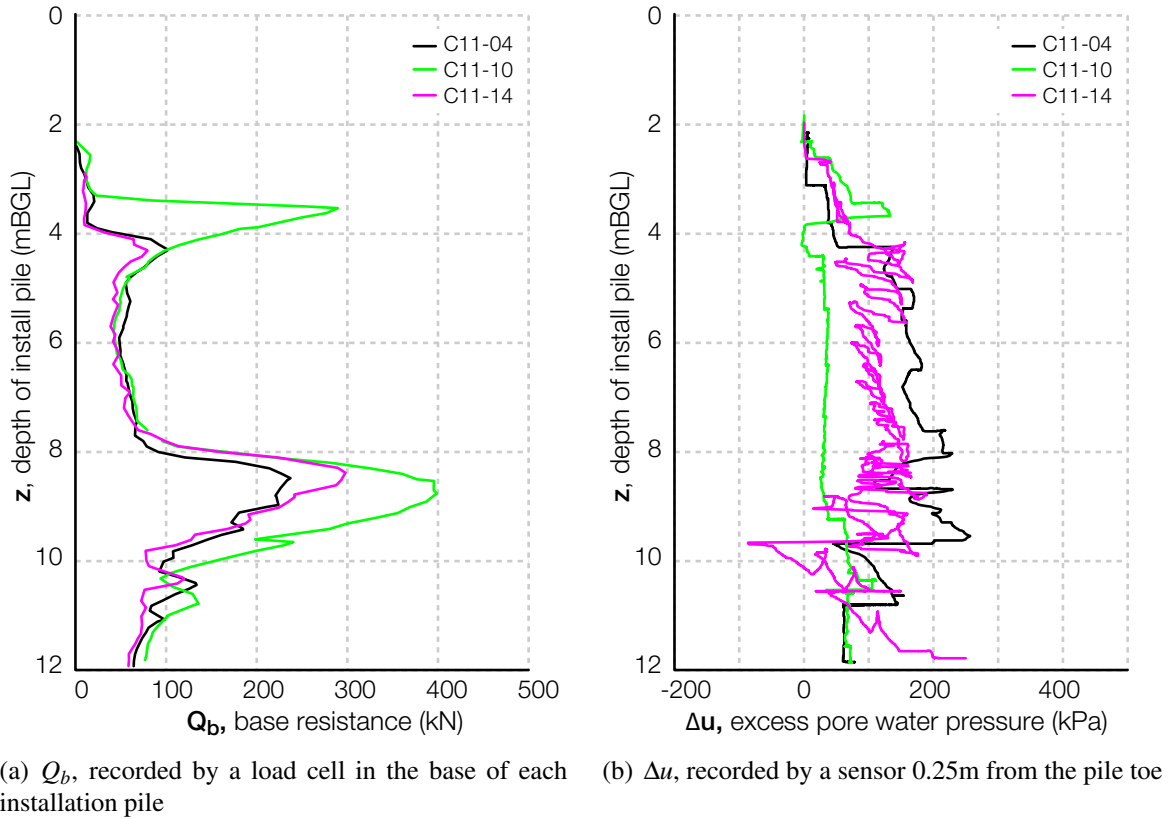


Figure 22: Comparison of base resistance, Q_b , and excess pore water pressures, Δu , during fast installation tests

The trends in Q_b and Δu for the fast installation cases have been included in Figures 22(a) and 22(b), and show similar behaviour to that seen in the slow installation cases.

Base resistances

It can be seen that the gyro-surfing technique (C11-14, pink line) experienced the smallest base resistance over the majority of the installation, except for at the peak, where the axial installation (C11-04, black line) encountered an unexpectedly low resistance. Consequently, it was the rotary-jacked installation (C11-10, green line) that recorded the maximum value of $Q_{b,peak}$ (~ 400 kN, $z = 8.8$ mBGL).

Excess pore water pressures

Unlike the slow cases, the excess pore water pressures Δu in the fast installations do not drop away to negligible values between depths of ~ 8 mBGL and 10 mBGL. The axially-jacked installation recorded the greatest values of Δu , recording a maximum value of 0.25 MPa at a depth of 9.5 mBGL. The excess pore water pressures generated during the gyro-surfed installation were

approximately 70% of those generated by the axially-jacked case.

Continuing problems with the pore water pressure sensor seem to have affected the readings taken during the rotary-jacked installation (C11-10), and the lack of variation over the installation is presumed to be anomalous behaviour.

5.2 Interpretation

A discussion of rates and “undrained” behaviour

From the results, it can be concluded that the base resistance in the silty sand is typically reduced in the presence of large excess pore water pressures. In slow installations ($v_z \approx 25\text{mm/s}$), it was not possible to generate excess pore water pressures in this layer, and peak base resistances of the order $\sim 375\text{kN}$ were recorded. By comparison, during fast installation cases ($v_z \approx 115\text{mm/s}$) it was possible to maintain positive excess pore water pressures through the silty sands a reduction in base resistance was observed. Values of $Q_{b,peak}$ as low as $\sim 230\text{kN}$ were recorded.

This conclusion is consistent with the theoretical understanding of “rate effects”, and with previous results from testing at Nunoshida (Delano, 2010; Bond, 2011). The significance of rate effects on installation has been further explained in Section 5.2.1

In the clay layers (between depths of ~ 4 and 8mBGL) positive excess pore water pressures are generated, almost independently of speed. Consequently, it is believed that the jacked installation through the clay layers behave in an “undrained” manner (a conjecture supported by the research of Deeks, 2008).

Benefits of gyro-surfing

One further observation worthy of discussion are the comparative benefits of different installation techniques. In each installation, it was the presence of highly permeable silty sand between depths of $\sim 8\text{mBGL}$ and 10mBGL that presented the greatest resistance to penetration. Not all the results are shown, but the installation technique that proved to be most consistent and effective at generating positive excess pore water pressures in this layer was gyro-surfing, installed with a resultant rate of $v = 112.2\text{mm/s}$. As a consequence of generating large excess pore water pressures, gyro-surfing experienced an average reduction in $Q_{b,average}$ of $\sim 14\%$.

The benefits resulting from gyro-surfing are best explained by rate effects: the combined effect of a fast axial install rate, and rotational velocity generate large excess pore water pressures under the base, and around the shaft of the pile, helping to reduce the resistance to penetration. The effect of surfing, as shown by Delano (2010) and Ogawa et al. (2011), helps to generate further excess pore water pressures around the base and shaft of the pile, further alleviating the resistance against penetration.

5.2.1 Rate effects

It has been shown from the results that increasing the rate of installation helps to reduce the base resistance acting on the pile. This is best exemplified by the axially-jacked test cases, where a reduction in $Q_{b,peak}$ of $\sim 65\%$ was observed between the fast installation cases ($v_z = 110\text{mm/s}$) and the slow installation cases ($v_z = 23\text{mm/s}$).

It was shown in Section 2.4 that the penetration resistance is closely related to the drainage conditions of the soil. At 8.5mBGL, where $Q_{b,peak}$ was typically encountered, the soils are silty sands, with an assumed permeability, c_v , of $10^5 \text{ m}^2/\text{year}$ (Bond, 2011). Due to the speed of installation, it is expected that both the fast and slow cases will behave in an approximately undrained manner.

Figure 23 lends credence to the hypothesis that the soils are behaving in an undrained manner. The results from axial testing (normalised by the results previously obtained by Bond, 2011) fall close to the correlation for the silty sands at Nunoshida, and demonstrate only small increases in base resistance when compared with the undrained case.

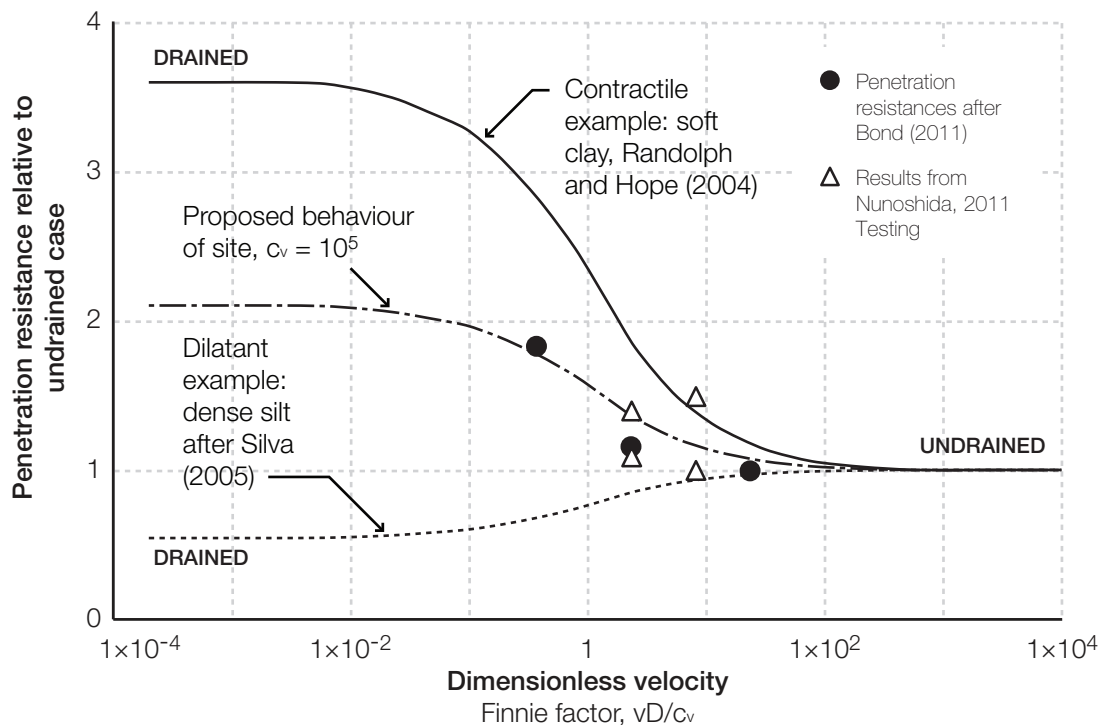


Figure 23: Evidence supporting the undrained behaviour of soils during press-in at Nunoshida

6 Developing a model for excess pore water pressure distribution around an installation pile

One of intentions of this report, is to develop a simple model for the distribution of pore water pressures around an installation pile. It is hoped that cavity expansion analyses, which have shown relevance to the prediction of pile bearing capacity, will be an appropriate starting point for this report.

The cylindrical and spherical cavity expansion models, based on the equations of Cao et al. (2002), has been presented in the section and are tailored to the conditions at Nunoshida. This gives the opportunity to calibrate and compare the model with real test data, which will confirm whether it is suitable for the prediction of pore water pressures around a pile.

6.1 Assumptions and input variables

6.1.1 Non-linear elastic/perfectly plastic soil

In Section 2, a model of non-linear elastic/perfectly plastic soil was presented, along with the recent research of Vardanega et al. (2012). It was concluded that the behaviour of real undrained cohesive soils correlates well this idealised behaviour, and is thus particularly suited to the clayey soils at Nunoshida, between depths of 4mBGL and 8.5mBGL.

6.1.2 Undrained conditions

It is well known that, to assume undrained conditions, it necessary to maintain a fast installation speed, such that Δu generated during the installation process does not have sufficient time to dissipate. Although it is difficult to simulate fully undrained behaviour in real pile installation, it was shown in Section 5.2.1 that the soils at Nunoshida, when subjected to a press-in installation, behave in a quasi-undrained manner. For the purpose of simplicity, undrained behaviour is thus assumed.

6.1.3 The generation of excess pore water pressures

In Section 2.3, it was shown that the generation of pore water pressures in an undrained non-linear elastic/perfectly plastic soil arose as a consequence of two primary features:

- In isotropic non-linear elastic soils, shearing causes a change in Δp , which, by virtue of the undrained conditions ($\Delta p' = 0$), causes a direct increase in Δu .

- The critical state model (Schofield and Wroth, 1968) showed that the plastic yielding of soil is shown as a left-wards trend in a plot of $q - p'$. From Terzaghi (1943), it can be shown that Δu will therefore increase by a factor of $\Delta p'$.

For simplicity, the generation of Δu from critical state yielding is ignored, as the remote sensors are expected to be predominantly outside the plastic regime (shown later in Table 4). Understanding the distribution of Δu in the plastic regime would require further testing of the in-situ soils in order to build an appropriate constitutive model.

6.1.4 The “pressure bulb”

One final simplification is assume that the distribution of stresses and pore water pressure are concentrated in a “pressure bulb” around the pile base.

Previous studies (e.g. Randolph et al., 1979) have concluded that the displacement of soil around the base of a pile installation is quasi-spherical, giving rise to “pressure bulb” behaviour, as described in Section 2.2. As such, it is believed that spherical cavity expansions (illustrated in Figure 24) will offer the best insight into the distribution of u around an installed pile.

With little additional computational effort, the equations of Cao et al. (2002) can also provide a cylindrical cavity expansion solution. For comparison, both spherical and cylindrical models shall be considered in this section.

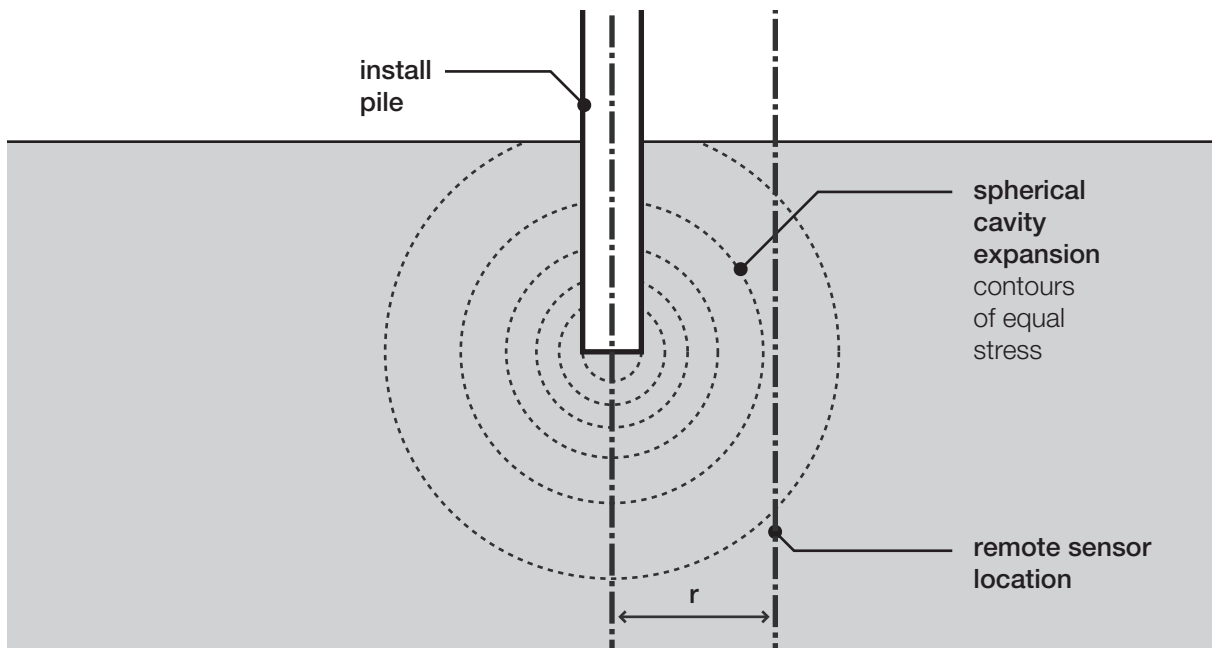


Figure 24: The idealised spherical cavity expansion at the base of an install pile

6.2 Calibrating the model with soil properties

During testing, the remote sensors installed in the clayey layer, were positioned at depths of 6mBGL and 7.5mBGL. The consistency of the soil behaviour at Nunoshida (as documented in Section 4) lends confidence to the quality of the data relating to the soil properties. As such, the spherical and cylindrical models, have been calibrated using soil properties at these respective depths (Table 3).

Table 3: Soil properties at 6mBGL and 7.5mBGL

Soil property	Value of soil property at 6mBGL (equivalent to depth of sensor PWP5)	Value of soil property at 7.5mBGL (equivalent to depth of sensor PWP4)	Source / Derivation
Undrained shear strength, c_u (kPa)	82.5kPa	133.0kPa	From CPT borehole data for undrained soil shear strength, presented in Figure 16
Mobilisation strain, $\gamma_{M=2}$	0.5%		Assuming soils have similar properties to Kaolin, (Vardanega et al., 2012)
β	0.5		After Bolton and Whittle (1999)
Strain at yield, γ_y	2%		Using Vardanega et al.'s modified constitutive equation, (Equation 2)
Unit weight of soil, γ	17kN/m ³		Assumed
Unit weight of water, γ_w	10kN/m ³		Assumed
K_0	1		Assumed
σ'_{r0} (σ_{r0})	42.0kPa (102.0kPa)	52.5kPa (127.5kPa)	$\sigma'_{r0} = K_0 \gamma' z$ ($\sigma_{r0} = K_0 \gamma' z + \gamma_w z$)

6.3 Preliminary calculations: the “plastic radius,” and corresponding soil stresses

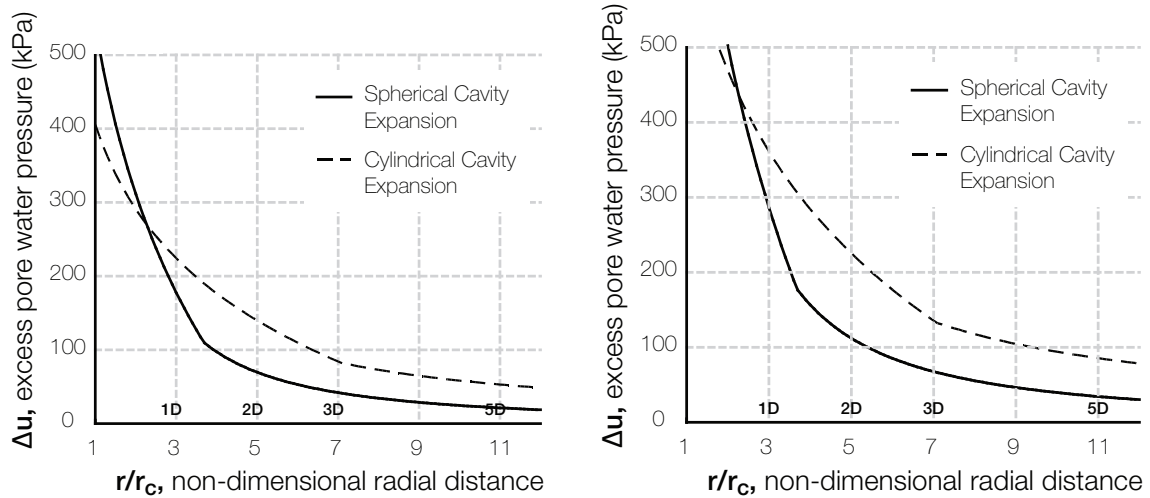
As a preliminary to developing a complete model for the distribution of stresses around a cavity expansion, it is necessary to calculate the location of the “plastic radius,” r_y and the corresponding stresses at that location.

The plastic radius, and the corresponding soil stresses expected at Nunoshida, can be calculated using the theory devised by Cao et al., whose equations are included in Appendix II. The results from these calculations have been summarised in Table 4.

Table 4: Summary of results from spherical and cavity expansion analyses, determining the “plastic radius” and corresponding soil stresses

	Spherical Cavity Expansion		Cylindrical Cavity Expansion	
	Magnitude at 6mBGL	Magnitude at 7.5mBGL	Magnitude at 6mBGL	Magnitude at 7.5mBGL
r_y (m)	0.58		1.13	
σ_{ry} (kPa)	322	267	482	394
$\sigma_{\theta y}$ (kPa)	157	-	216	-
u_y (kPa)	150	185	232	261

6.4 The distribution of stresses throughout the foundation soil



(a) Cylindrical and spherical cavity expansions in soils at a depth of 6mBGL

(b) Cylindrical and spherical cavity expansions in soils at a depth of 7.5mBGL

Figure 25: Distribution of Δu around a cavity in soils at 6mBGL and 7.5mBGL

The combination of the assumptions and results presented in Sections 6.1 – 6.3 provides sufficient information to make a model of cylindrical and spherical cavity expansions in the clayey soils at Nunoshida.

The theoretical results from the model illustrating the radial distribution of Δu around a cavity expansion have been presented in Figure 25. The results from the cylindrical cavity expansion plot are shown as a black dotted line, whilst the results from the spherical cavity expansion plot are shown as a solid black line. These theoretical trends are compared with field data from the Nunoshida test installations in the following section (Section 8).

7 The radial distribution of excess pore water pressures (Δu)

The simple model presented in Section 6 is intended to predict the distribution of excess pore water pressures around an installation pile. From the tests conducted at Nunoshida, a wealth of high-quality data collected by the remote pile is available to verify the radial distribution predictions of the model.

This “wealth” of data, describing the distribution of pore water pressures, takes the form of 160 individual data streams, or 13,319,520 data points. Thus it is important to identify and extract the salient properties from this data bank that will represent short- and long-term pore water pressure behaviour. This forms the focus for this section.

7.1 Initial data processing

The radial distribution of Δu was recorded by pore water pressure sensors fixed to remote monitoring piles, and some initial results of Δu , are shown in Figure 26(b).

The “peaks” in the data are of great interest, but in their raw form, they are difficult to interpret. Thus, it was decided to align the peaks by defining five new “relative” axes — one for each pore water pressure sensor. The axes were defined using the formula, $z_{rel} = z_i - z$, where z and z_i are the installation depth, and the depth of the remote sensor respectively (Figure 26(a)). The results of data processing can be seen in Figure 26(c).

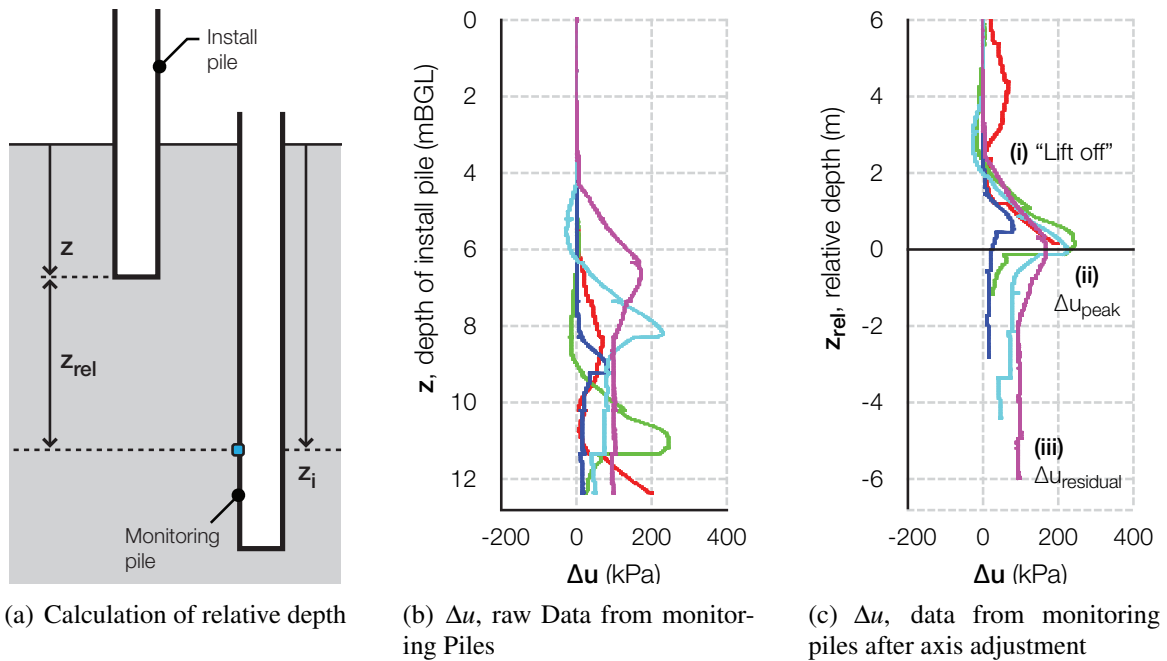
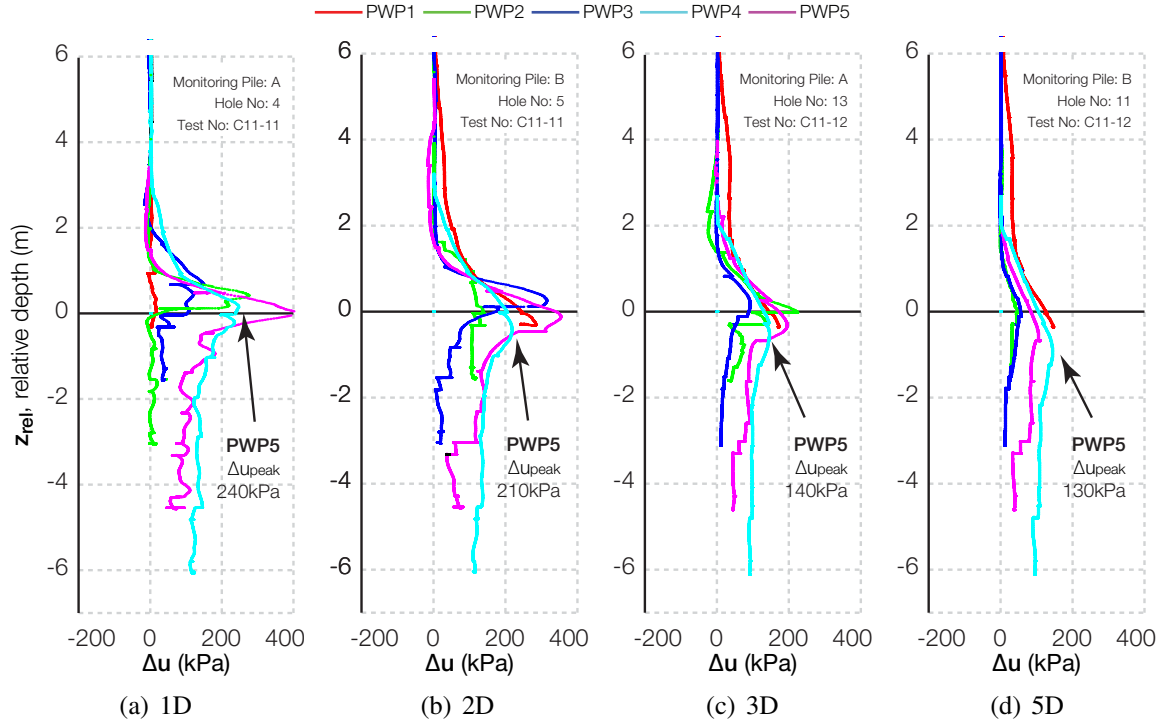


Figure 26: Processing the raw excess pore water pressure data: redefining the axes

Figure 27: Radial variation of Δu

7.2 Results

The “typical” trend in excess pore water pressures, as illustrated in Figure 26(c), was consistently experienced across all test installations. Further plots of Δu , included in Figure 27, help to illustrate this consistency, and also show the variation of pore water pressures with radius. These trends and radial variation in Δu are discussed in Sections 7.2.1 – 7.2.2.

7.2.1 Salient features: the rise and fall of Δu

In each of Figures 27(a) – 27(d), the following trend can be seen:

- The excess pore water pressure is initially zero ($\Delta u_0 = 0$).
- As the base of the pile penetrates into the ground, the pore water pressure rises. The sensor records a change in pore water pressure (the point of “lift off”) when the base of the advancing pile is at a relative distance of $\sim 1.9\text{m}$ (6 diameters) above the horizon of the sensor (i.e. $z_{rel} = +6D$).
- The excess pore water pressure reaches a peak value (Δu_{peak}) when the installation pile is approximately level with the remote sensor.

- After the install pile has passed through the horizon of the sensor, the excess pore water pressure drops.
- The final excess pore water pressure, defined as the “residual excess pore water pressure” ($\Delta u_{residual}$), is typically positive — i.e. greater than Δu_0 . The residual value is typically reached after the pile has advanced a further 1.9m (6D) beyond the horizon of the pore water pressure sensor. The only exception to this are the trends produced by the pore water pressure sensor in the sand layer (PWP3), shown as the blue line in Figure 27, whose residual value is negligible (i.e. $\Delta u_{PWP3,residual} \approx 0$)

7.2.2 The radial variation of Δu

From Figure 27, it should be noted that the magnitude of Δu_{peak} decreases with increasing radius. As an example, PWP5, a “monitoring” sensor positioned in clayey soils at a depth of 6mBGL, recorded a peak excess pore water pressure of approximately 240kPa during the 1D test installation (Figure 27(a)). Under the same conditions, a sensor positioned at 5D recorded a magnitude of $\Delta u_{peak} = 130\text{kPa}$ (Figure 27(d)). This represents a reduction of $\sim 45\%$.

7.3 Interpretation

7.3.1 Justification for rise and fall of Δu — the passing pore water “pressure bulb”

It is encouraging to notice that the theoretical discussions of “pressure bulbs” (Section 2.2) has been observed in the field test results. This also lends confidence to the assumptions being made with regards to spherical cavity expansion behaviour.

The results show that, as an installation pile is pressed into the soil, a remote sensor will first detect the very edge of the pressure bulb, recording a small increase in excess pore water pressure. As the pile base gets ever closer, in the vertical plane, to the monitoring sensor, the stresses recorded by that sensor will increase. After the pile has passed, the distance between the applied load and the sensor increases, and the soil relaxes. All these observations are consistent with spherical cavity expansion and pressure bulb theories.

7.3.2 The peak values of Δu

From the point-of-view of data processing, it is easy to identify Δu_{peak} . Furthermore, Δu_{peak} has great significance and importance, for it is believed to be representative of the pressure bulb / spherical cavity expansion mechanism. As such, further analysis in comparing Δu_{peak} to the theoretical model shall be presented in Section 8.

7.3.3 The residual values of Δu

The presence of $\Delta u_{residual}$ is of great importance, for two main reasons:

- $\Delta u_{residual}$ supports the assumptions that the soil is undrained. During testing, the slow installations lasted approximately 20 minutes. In the silty soils, with an assumed permeability of c_v , of $10^5 \text{ m}^2/\text{year}$ (Bond, 2011), it is expected that the installations will be fast enough to simulate undrained conditions. The presence of $\Delta u_{residual}$ supports this hypothesis.
- The long-term effects, such as set-up are likely to be closely linked to the value of $\Delta u_{residual}$. As these residual pressures dissipate, the soil will regain effective stresses, and a corresponding increase in pile capacity will be observed (Jackson, 2007).

However, the presence of $\Delta u_{residual}$ is believed to be difficult to explain using the simple spherical cavity expansion hypothesis. The removal of the applied load, with the passing of the pile, results in a relaxation of the soil. However, there still remains a deformation of some kind – it does not return to its original state. Consequently, the behaviour that is observed is believed similar to the unload cycle of a pressuremeter (e.g. Bolton and Whittle, 1999), where the non-linearity of the soil displays a hysteresis-style of behaviour. An indicative illustration of this relaxation process has been plotted on stress-strain axes in Figure 28.

The exact behaviour will be reflective of the critical state conditions developed by the soil during plastic deformation, and could potentially be modelled by finite element analysis, with an appropriate constitutive model. However, this analysis falls beyond the scope of the report.

Instead, the experimental values will be taken forward for further analysis in Section 8, where empirical correlations between Δu_{peak} and $\Delta u_{residual}$ shall be considered.

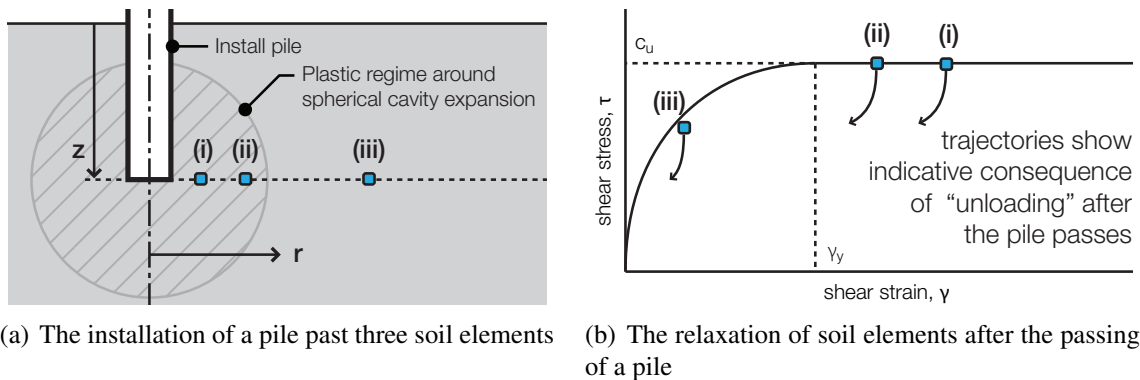


Figure 28: Indicative relaxation behaviour after the passing of a pile

8 Comparing cavity expansion analysis with the field data

In Section 6, a model for cavity expansion in an undrained non-linear elastic/perfectly plastic soil was developed. To verify the suitability of the cavity expansion model for its desired application, it is necessary to compare it with field data. Section 7 presented field data showing the behaviour of soils around a typical pile installation, and identified Δu_{peak} and $\Delta u_{residual}$ as key features of the installation process. The comparison of the theoretical model with Δu_{peak} and $\Delta u_{residual}$ has been presented in this section.

An empirical correlation between Δu_{peak} and $\Delta u_{residual}$ has also been prepared. This correlation can potentially be used to determine the link between short- and long-term effects of pore water pressures.

8.1 Results

8.1.1 Peak excess pore water pressures (Δu_{peak})

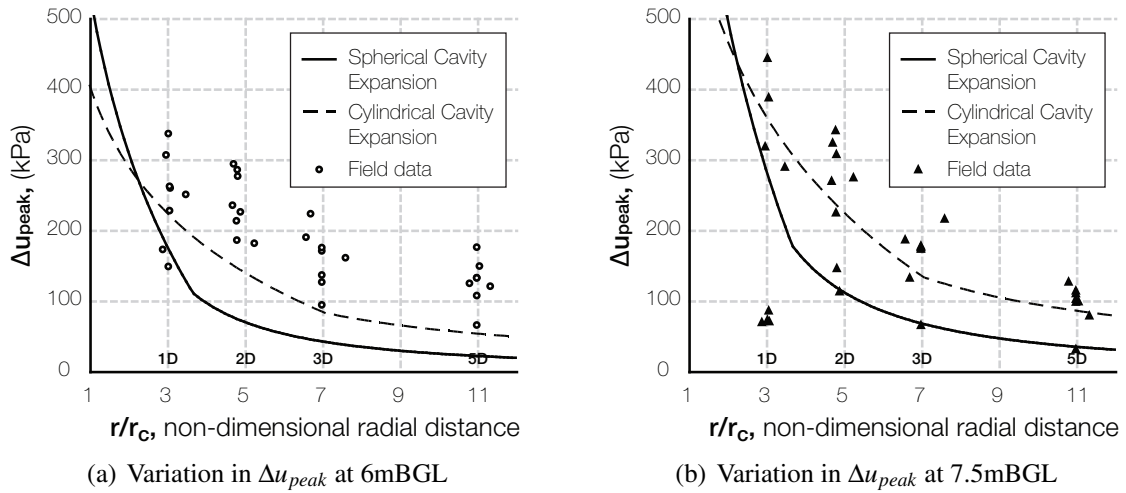


Figure 29: The variation in Δu_{peak} with radius

The “peak” excess pore water pressures, recorded by sensors on the monitoring piles at 6mBGL and 7.5mBGL, have been plotted in Figures 29(a) and 29(b) respectively. The results from all 16 installations have been shown. The largest values of Δu_{peak} are recorded by the sensors closest to the installation pile. The general trend showed a decrease in Δu_{peak} with increasing radius. In both plots, it is noted that the spherical cavity expansion analysis forms a good estimate of the lower bound.

8.1.2 Residual excess pore water pressures ($\Delta u_{residual}$)

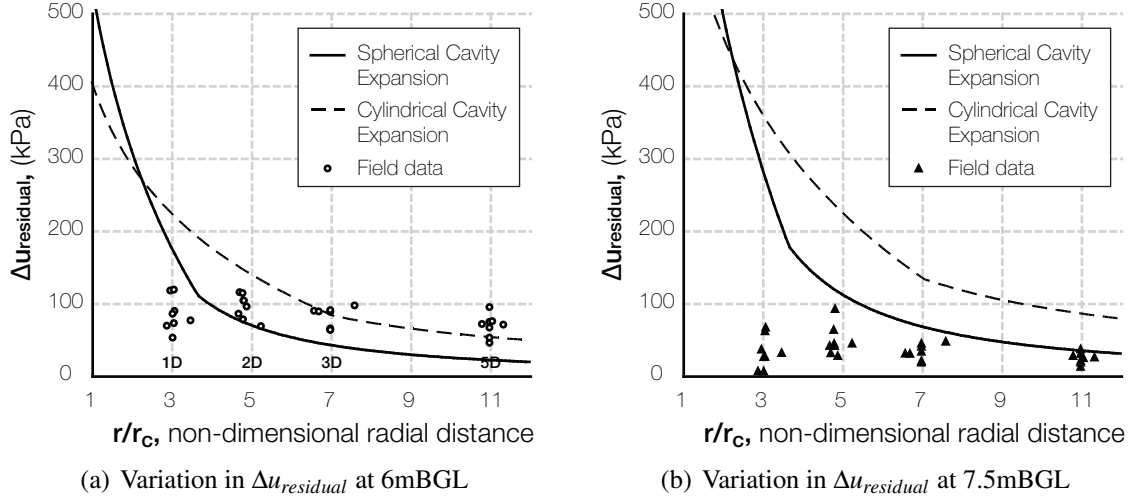


Figure 30: The variation in $\Delta u_{residual}$ with radius

Figures 30(a) and 30(b) display the residual excess pore water pressures generated during all 16 pile installations, as recorded by sensors positioned at depths of 6mBGL and 7.5mBGL respectively. By comparison with the plots for peak excess pore water pressure, the radial variation in the results is almost negligible. Furthermore, the spread of values recorded are also much smaller. In Figure 30(a), all the excess pore water pressures are within the range 50kPa – 120kPa.

8.2 Interpretation

8.2.1 The suitability of spherical cavity expansion analysis for predicting Δu_{peak}

It is pleasing to note that the predictions and assumptions made in developing a model for spherical cavity expansion produces a good estimate of the lower bound for Δu_{peak} .

During the development of the spherical cavity expansion model, the most significant assumption made, was to ignore the $\Delta p'$ effect described by the critical state method of Schofield and Wroth. From the results, it appears that, under the undrained shearing of soils at Nunoshida during installation, the $\Delta p'$ effect can generate up to 250kPa of additional excess pore water pressure, when compared with the simple non-linear elastic/perfectly plastic assumptions of Bolton and Whittle.

In order to better predict the value of Δu_{peak} generated during pile installation, additional development of the simple spherical cavity expansion model would be recommended. This

could be achieved through empirical testing, or by finite element analysis, with an appropriate constitutive model.

8.2.2 Analysis of residual excess pore water pressures ($\Delta u_{residual}$)

Across all tests, the value of $\Delta u_{residual}$ was considerably smaller than that of Δu_{peak} . The hypothesis put forward in Section 7.3.3 justified the drop in pore water pressures between the peak and residual values by using the analogy of unloading a pressuremeter. This hypothesis appears to be an acceptable justification — the hysteresis resulting from undrained relaxation would be recorded as a decrease in pore water pressure.

Hysteresis also justifies the observation that there is little variation in $\Delta u_{residual}$ with radius. In the near field, deformation mechanism around the pile base exerts large stresses on the surrounding foundation soil. Further away in the far-field, these are less pronounced. Consequently, the removal of this load due to the passing of the pile base will yield a larger relaxation through hysteresis in the near-field than the far-field. In undrained conditions the pore water pressures will record a corresponding change, with the most pronounced drop being experienced in the near-field, and thus the radial variation will also be less obvious.

Due to the difficulties of quantifying the effect of hysteresis through the simple models developed in this report, an empirical comparison of peak and residual pore water pressures is included below.

8.3 Comparison of peak and residual pore water pressures

An analysis comparing the magnitude of peak and residual pore water pressures, shows there is an approximately linear correlation between the two (Figure 31). The results shown have been taken from the remote sensors PWP3 (9mBGL), PWP4 (7.5mBGL) and PWP5 (6mBGL) from all 16 tests, and thus represent every different type of installation procedure. To account for effects of different installation procedures, a non-dimensional vertical axis has been used — the peak excess pore water pressure has been normalised by the instantaneous value of base pressure corresponding to the point in time when the remote pressure was a maximum.

It should be noted that data from remote sensors PWP1 and PWP2 have been omitted from this analysis. As mentioned in Section 7.2, the trends in excess pore water pressure typically find their residual value after the installation pile has advanced a further 1.9m (6D) past the horizon of the remote sensor. In the cases of PWP1 and PWP2, the pile never advanced this far, and as such, no obvious residual value was detected in their respective trends.

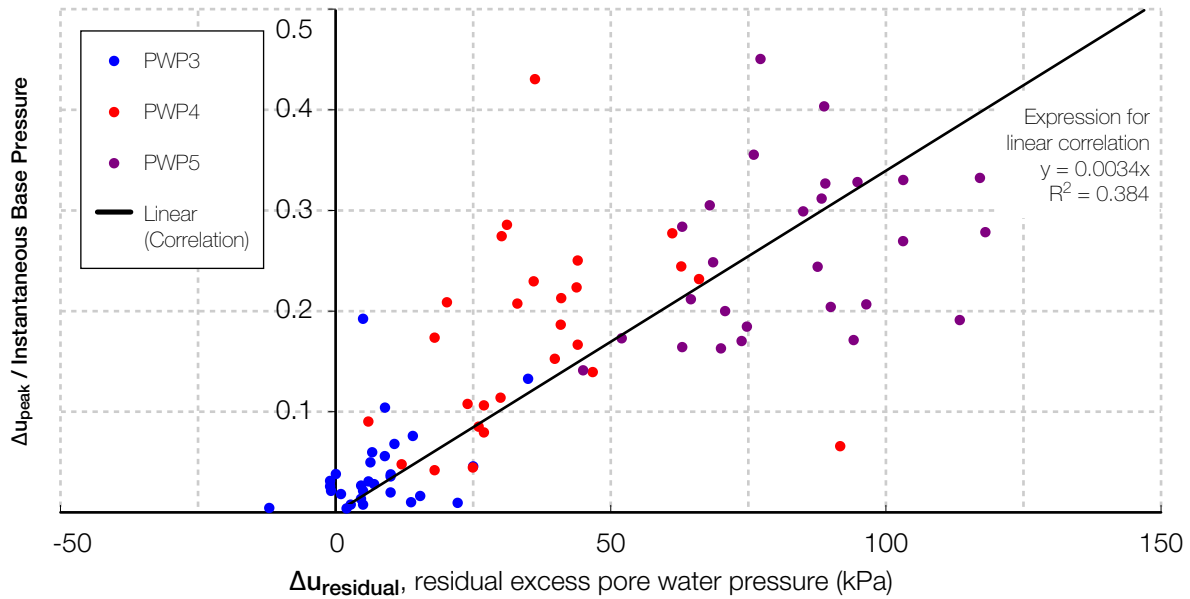


Figure 31: Comparison of non-dimensional peak and residual excess pore water pressures

8.3.1 Interpretation of the correlation between peak and residual excess pore water pressures

Consideration of PWP3 (9mBGL)

The data points relating to remote sensor PWP3 were grouped around the origin – an outcome which was expected. PWP3, installed at a depth of 9mBGL, is located in a sand layer. Sand presents a high resistance to the install pile, and pore water pressures are quick to dissipate due to the high permeability of the soil. Consequently, the normalised peak excess pore water pressure, and the residual values are likely to be relatively small.

Consideration of PWP4 & PWP5 (7.5mBGL & 9mBGL)

PWP4 and PWP5 are both located in clayey soils, and exhibited undrained behaviour. They were able to generate large considerable Δu during installation, helping to reduce Q_b , and leading to the generation of considerable residual pore water pressures.

The specific values recorded by PWP4 and PWP5 are slightly different, with PWP5 typically recording larger values of normalised peak and residual excess pore water pressure, when compared with PWP4. This can be explained by their different soil types. PWP4 is positioned at 7.5mBGL in a stratum of clayey sand, whilst PWP5 is positioned at 6mBGL in a stratum of sandy clay. PWP4 will be in a soil stratum with higher strength and higher permeability, and will consequently record lower values of normalised peak and residual excess pore water pressure than PWP5.

8.4 Summary and recommendations for further research

The two correlations presented in this section have the potential for a practical application. The spherical cavity expansion model, and the non-linear elastic/perfectly plastic soil idealisation, offer a good lower bound estimate for the magnitude of Δu_{peak} around a pile installation in the soils at Nunoshida. The linear correlation presented in Figure 31 draws a reasonable link between Δu_{peak} and $\Delta u_{residual}$. It could thus be imagined, that, when used together, predictions for $\Delta u_{residual}$ (and hence long-term effects such as set-up) could be based on a spherical cavity expansion analysis. This concept, however, is very much a hypothesis, and would require significant additional research.

Initial thoughts for further research

For practical purposes, it is recommended that considerations should first focus on different soil types and different drainage conditions. A hypothesised graph, based on the results of Randolph (2004) and Silva et al. (2005) for different isotropic soils types, and different drainage conditions has been thus been included for illustrative purposes in Figure 32.

In contractile soils, positive excess pore water pressures are generated with increasing installation rate. The undrained case will resemble the results at Nunoshida, recording large non-dimensional peak pressures *and* large residual excess pore water pressures, resulting in an gradient similar to that shown in Figure 31. However, with higher permeability soils and slower installation speeds, the corresponding Finnies factor will drop, and installation will behave in an undrained manner and consequently, after installation, there will be negligible $\Delta u_{residual}$. These results would fall on a line of steeper gradient.

In dilatant soils (highlighted as the bottom left-hand quadrant), Silva et al. (2005) noted the generation of negative Δu during slow, drained tests. However, the magnitude of these negative pore water pressures are likely to be small, and likely to be clustered around the origin.

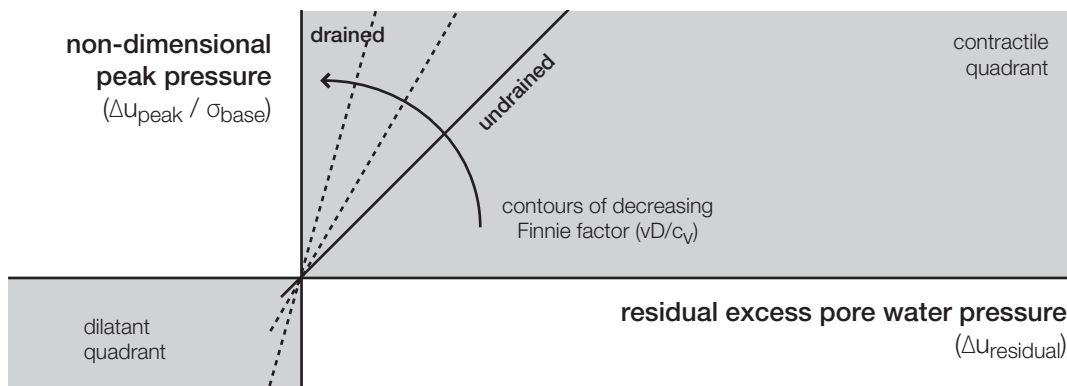


Figure 32: Hypothesised correlation between Δu_{peak} and $\Delta u_{residual}$ for different soil types.

9 Conclusions

The purpose of this report was to highlight the significance of pore water pressures on press-in piles. A particular focus was chosen to investigate the effect of excess pore water pressures on the installation process. To assist with this research, a simple theoretical cavity expansion analysis was developed, simulating a pile installation. Four weeks of field testing were also completed in order to support the project aims. The following conclusions have been proposed:

- A simple analytical model of a spherical cavity expansion in a undrained, non-linear elastic/perfectly plastic soil can be prepared, provided relevant information regarding the soil properties are available. The combination of CPT and PPT tests, were able to offer appropriate information for development of a spherical cavity expansion model.
- Press-in field tests were conducted in Kochi, Japan, on a site comprising normally consolidated alluvial deposits. In cohesive layers, the soil responded in an undrained manner, and positive excess pore water pressures were observed throughout the foundation soil. The furthest monitoring sensors, positioned at 11 pile radii from the centre of the installation pile also noticed this effect. These effects were also consistently observed in fast installations into sandy soils.
- For slow installations in sandy layers, the soils behaved in a drained manner, and pore waters were dissipated rapidly during the installation. Consequently, it was difficult to generate excess pore water pressures, and negligible values were recorded at most radii.
- From the results, the presence of a “pressure bulb” acting around the base of the install pile was noted. Sensors positioned at various radii recorded the passing of the pressure bulb as a peak (Δu_{peak}), typically occurring when the base of the pile was on the same horizon as the sensor. Subsequent relaxation saw the value of excess pore water pressure fall to a residual value ($\Delta u_{residual}$)
- A comparison between the theoretical model and the field data showed that an undrained expansion of a spherical provides a good estimate of the lower bound of Δu_{peak} .
- An empirical correlation between Δu_{peak} and $\Delta u_{residual}$ was found to be approximately linear.
- Further research is required to validate or improve the models and mechanisms proposed in this report. Preparing a design methodology to link the predictions of short-term pore water pressure to long-term effects (e.g. set-up) would be of particular interest to the author.

References

- M. D. Bolton and R. W. Whittle. A non-linear elastic/perfectly plastic analysis for plane strain undrained expansion tests. *Géotechnique*, 1999.
- M. D. Bolton, H. W. Sun, and A. M. Britto. Finite element analyses of bridge abutments on firm clay. *Computers and Geotechnics*, pages 221–245, 1993.
- M.D. Bolton and Y.P. Cheng. Micro-geomechanics. In *Proceedings of the Workshop on Constitutive and Centrifuge Modelling: Two Extremes*, pages 59–74, Monte Verità, Switzerland, 2002.
- T. Bond. Rotary jacking of tubular piles. Master’s thesis, University of Cambridge, 2011.
- S. Boyle. The effect of piling operations in the vicinity of computing systems. *Ground Engineering*, pages 23–27, 1990.
- R. Butterfield and P. K. Banerjee. The effect of pore water pressures on the ultimate bearing capacity of driven piles. In *Proceedings of the Second South East Asian Conference on Soil Engineering*, pages 385–394, Bangkok, Thailand, 1970.
- L. F. Cao, C. I. Teh, and M.-F. Chang. Analysis of undrained cavity expansion in elasto-plastic soils with non-linear elasticity. *International Journal for Numerical and Analytical Methods in Geomechanics*, 2002.
- F. C. Chow. *Investigations into the behaviour of displacement piles for offshore foundations*. PhD thesis, University of London (Imperial College), 1997.
- J. I. Clark and G. G. Meyerhof. The behaviour of piles driven in clay. an investigation of soil stress and pore water pressure as related to soil properties. *The Canadian Geotechnical Journal*, 9(3), 1972.
- K. W. Cole. Uplift of piles due to driving displacement. *Civil Engineering and Public Works Review*, pages 263–269, 1972.
- J. H. A. Crockett. Piling vibrations and structural fatigue. In *Recent Developments in the Design and Construction of Piles*, pages 305–320. The Institution of Civil Engineers, 1980.
- A.D. Deeks. *An investigation into the strength and stiffness of jacked piles in sand*. PhD thesis, University of Cambridge, Cambridge, 2008.
- O. Delano. The application of surging on jacked-in piles. Master’s thesis, University of Cambridge, 2010.
- R. E. Gibson and W. F. Anderson. In situ measurement of soil properties with the pressuremeter. *Civil Engineering Public Works Review*, 1961.
- S. S. Gue. *Ground heave around driven piles in clay*. PhD thesis, University of Oxford, Oxford, UK, 1984.

- M. J. Gunn. The prediction of surface settlement profiles due to tunnelling. In *Proceedings of the Wroth Memorial Symposium*, pages 304–316, Oxford, England, 1992.
- D. J. Hagerty and R. B. Peck. Heave and lateral movements due to pile driving. *Journal of the Soil and Foundations Division, Proceedings of the ASCE*, pages 8497–8517, 1971.
- J. M. Head and F. M. Jardine. Ground-borne vibrations arising from piling. Technical report, CIRIA Technical Note 142, 1992.
- A. R. House, J. R. M. S. Oliveira, and M.F. Randolph. Evaluating the coefficient of consolidation using penetration tests. *International Journal of Physical Modelling in Geotechnics*, 2001.
- Y. Ishihara, K. Okada, M. Nishigawa, N. Nogawa, Y. Horikawa, and A. Kitamura. Estimating ppt data via cpt based design method. In *Press-in Engineering 2011: Proceedings of 3rd IPA International Workshop in Shanghai*, volume 1, pages 84–94, Shanghai, China, 2011.
- A. Jackson. The setup of jacked piles. Master’s thesis, University of Cambridge, Cambridge, 2007.
- A. Jackson. Pile jacking in sand and silt. 2008.
- R. J. Jardine, J. R. Standing, and F. C. Chow. Some observations of the effects of time on the capacity of piles driven in sand. *Géotechnique*, 56(4):227–244, 2006.
- B. M. Lehane and R. J. Jardine. Displacement pile behaviour in a soft marine clay. *Canadian Geotechnical Journal*, 31(1):181191, 1994.
- G. G. Meyerhof. The ultimate bearing capacity of foundations. *Géotechnique*, 2(4):301–332, 1951.
- P. W. Mitchell and J. Manders-Jones. Jacked piling in a soil subject to relaxation. *Australian Geomechanics*, 39(4):25–32, 2004.
- N. Ogawa, Y. Ishihara, M. Nishigawa, and A. Kitamura. Effect of surging in press-in piling. In *Press-in Engineering 2011: Proceedings of 3rd IPA International Workshop in Shanghai*, volume 1, pages 101–106, Shanghai, China, 2011.
- M. F. Randolph. Characterisation of soft sediments for offshore applications. In *Proceedings of the 2nd International Conference on Site Characterisation*, volume 1, pages 209–231, Porto, Portugal, 2004.
- M. F. Randolph, J. P. Carter, and C. P. Wroth. Driven piles in clay - the effects of installation and subsequent consolidation. *Géotechnique*, 29(4):361–393, 1979.
- M.F. Randolph, J. Dolwin, and R.D. Beck. Design of driven piles in sand. *Géotechnique*, 44(3):427–448, 1994.

- M. Roy, D. Michaud, and F. A. Tavenas. The interpretation of static cone penetration tests in sensitive clays. In *Proceedings of the European Symposium on Penetration Testing*, volume 2.1, pages 323–331, Stockholm, Sweden, 1975.
- A. N. Schofield and C. P. Wroth. *Critical state soil mechanics*. McGraw-Hill, New York, 1968.
- H. B. Seed and L. C. Reese. The action of soft clay along friction piles. *Transactions of the American Society of Civil Engineers*, 122:731754, 1957.
- F. S. Silva, D. J. White, and M.D. Bolton. An analytical study of the effect of penetration piezocone tests in clay. *International Journal for Numerical and Analytical methods in Geotechnics*, pages 501–527, 2005.
- M. M. Soares and C. R. R. Dias. Behavior of an instrumented pile in the rio de janeiro clay. In *Proceedings of the 12th International Conference on Soil Mechanics and Foundations in Engineering*, page 319322, Rio de Janeiro, 1989.
- L. O. Soderberg. Consolidation theory applied to foundation pile time effects. *Géotechnique*, 12(3):217–225, 1962.
- K. Terzaghi. *Theoretical Soil Mechanics*. John Wiley and Sons, Inc., 1943.
- P. J. Vardanega, B. H. Lau, S. Y. Lam, S. K. Haigh, S. P. G. Madabhushi, and M. D. Bolton. Laboratory measurement of strength mobilisation in kaolin: link to stress history. *Géotechnique Letters*, 2012.
- A.C. Vesic. Expansion of cavities in infinite soil mass. *Journal Soil Mechanics and Foundation Division, ASCE*, 98(SM3):265–290, 1972.
- D. J. White and M. D. Bolton. Displacement and strain paths during plane-strain model pile installation in sand. *Géotechnique*, 2004.
- D. J. White and A. D. Deeks. Recent research into the behaviour of jacked foundation piles. In *Proceedings of the International Workshop on Recent Advances in Deep Foundations*, Yokosuka, Japan, 2007.
- D. J. White, A. D. Deeks, and Y. Ishihara. Novel piling: Axial and rotary jacking. In *Proceedings of the 11th International Conference of the Deep Foundations Institute*, London, England, 2010.
- A. G. Yetginer, D. J. White, and M. D. Bolton. Field measurements of the stiffness of jacked piles and pile groups. *Géotechnique*, 56(5):349–354, 2006.
- L. Zeevaert. Discussion of “effect of driving piles into soft clay” by a. e. cummings et al. *Transactions of the ASCE*, 115, 1950.

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Appendix I: Health & Safety

Field testing was completed over a period of four weeks in Summer 2011, at the Nunoshida test site of Giken Seisakusho (Kochi, Shikoku, Japan).

Before testing, all members involved in site work were given a health and safety induction, and equipped with appropriate Personal Protective Equipment (PPE), in accordance with Health and Safety guidelines. An area of seismological activity, earthquake and tsunamis were thought to be significant risks. The location of tsunami shelters were made clear to all members in the work team, and the advice of the local authority was also made clear. It should be noted that site experimentation passed without any injury.

Since the completion of field testing, project work has been entirely computer-based. Due regard has been given to health and safety considerations regarding prolonged computer use. Mitigation against repetitive strain and eye-strain took the form of regular breaks and a suitable work environment. No health problems arose from completion of this project.

Appendix II: Key Equations for undrained cavity expansion, after Cao et al. (2002)

Elastic Loading

$$\sigma_r = p_0 + \frac{2m}{m+1} \frac{\tau}{\beta} \quad (3)$$

$$\sigma_\theta = p_0 + \frac{2m}{m+1} \frac{\tau}{\beta} - 2\tau \quad (4)$$

$$u = u_0 + \frac{2m\tau}{m+1} \left(\frac{1}{\beta} - 1 \right) \quad (5)$$

Values at the elastic-plastic interface

$$\sigma_{ry} = p_0 + \frac{2}{m+1} \frac{mc_u}{\beta} \quad (6)$$

$$\sigma_{\theta y} = p_0 + \frac{2}{m+1} \frac{mc_u}{\beta} - 2c_u \quad (7)$$

$$u_y = u_0 + \frac{2mc_u}{m+1} \left(\frac{1}{\beta} - 1 \right) \quad (8)$$

$$r_y^{m+1} = \frac{r_c^{m+1} - r_0^{m+1}}{1 - \exp[-\gamma_y]} \quad (9)$$

Plastic loading

$$\sigma_r = \sigma_{ry} + \frac{2mc_u}{m+1} \ln \left[I_r \left(\frac{r_c^{m+1} - r_0^{m+1}}{r^{m+1}} \right) \right] \quad (10)$$

$$u = u_y + \frac{2mc_u}{m+1} \ln \left[I_r \left(\frac{r_c^{m+1} - r_0^{m+1}}{r^{m+1}} \right) \right] + p'_0 \left[1 - \left(\frac{R}{2} \right)^\Lambda \right] \quad (11)$$

Note:

$m = 1$ for a cylindrical cavity and $m = 2$ for a spherical cavity

$I_r = G_y/c_u$ (the rigidity index)

$R = 2$ (the over-consolidation ratio of a elastic/perfectly plastic material)

$\Lambda = 1 - \kappa/\lambda$ (the plastic volumetric strain ratio)