Precise Surveying on Coe Fen

by

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Fourth-year undergraduate project in
Group D, 2011/2012

I hereby declare that, except where specifically indicated, the work submitted herein is my own original work.

Signed: ___________________________ Date: _____________

Contents

List of Figures

List of Tables

1 Introduction

Each year part IIA students in group D undertake a course(EAA) on the fields behind the Engineering department where they are taught the principles of conventional surveying. The fields are split into four areas (A-D) with each area having four stations. A station is a concrete block with a metal circle on its surface over which instruments can be placed and measurements made. Students are given initial coordinates for some of the stations to an accuracy of 1mm, from these they find the coordinates of the other stations and set out two additional points. However, in order to verify their results the location of the stations needs to be known as accurately as possible. The students measure the heights of all the stations by levelling from one end of the field to the other.

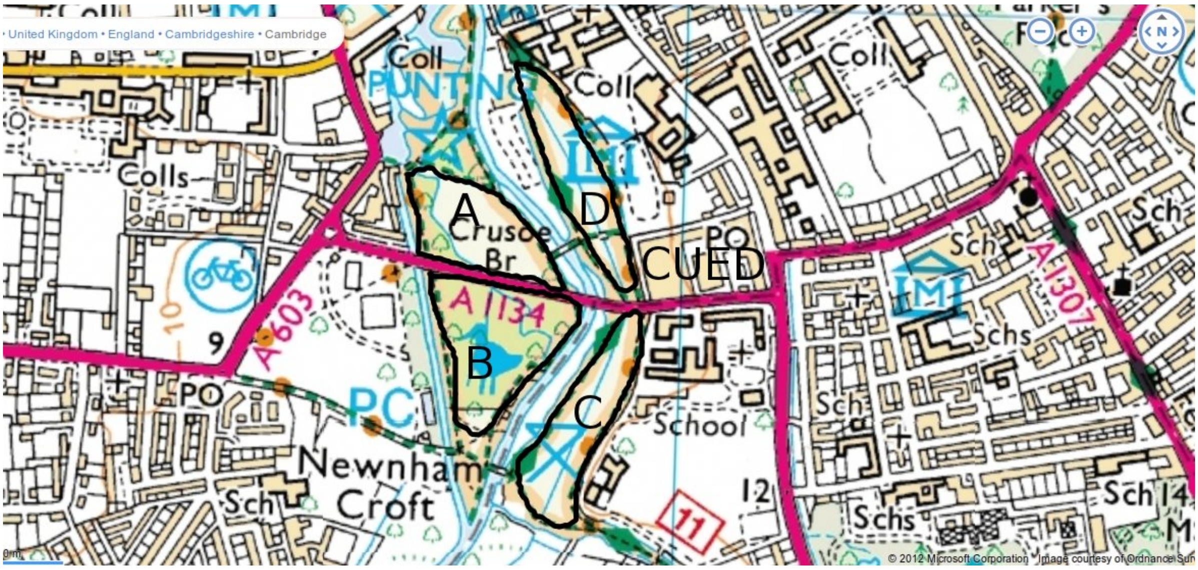


Figure 1: Map showing location of areas A-D in relation to the Engineering Department[1]

However due to the fact that the water table is not far below the grounds surface and the river Cam, which occasionally bursts its banks, runs through the area, many of the stations are susceptible to subsidence or uplift which will change their location. Growth of vegetation or construction work may lead to the lines of sight between station being obstructed which may warrant the relocation of a station, the new coordinates of which will need to be known. The stations may also be damaged through vandalism or vehicles working in the fields and so may need to be replaced. The aims of this project are to ensure that the coordinates of each station are known as accurately as possible. A transform was also to be produced to allow the conversion of GPS measurements to the local coordinate system that would produce better results in the local area than the Ordnance Survey transform.

There are also three distant reference objects located around Cambridge which are used to measure angles from (The Engineering Department chimney, the Southwest spire on King's College Chapel and the Roman Catholic Church on Hills road) The location of these objects is known to a reasonable degree of accuracy from previous years results but one of the objects of this project is to improve them.

2 Theory

Surveying is used to find the positions of points on the ground and the angles and distances between them. When a GPS system is use to find the location of a point on the earth, it is represented as a point on an ellipsoid that approximates the shape of the Earth. The point is defined by its latitude and longitude on the surface of the ellipsoid and also its height above (or below) the surface. However it is more useful if the locations can be represented on a two dimensional plane such as a map; in Britain the OSGB36 grid system is used. The following sections describe some of the principles used in GPS surveying and the process of converting coordinates from latitude and longitude to Eastings and Northings of OSGB36.

2.1 Geoid

A geoid is a theoretical surface covering the earth on which every point has the same gravitational potential energy. A geoid roughly follows the earths surface but rises or falls due to local variations in mass, such as mountain ranges or seas. There are an infinite number of possible geoids and in Britain the Newlyn Datum is used to define the British geoid. The Newlyn Datum is defined as the mean sea level between 1915 and 1921 in Newlyn, Cornwall[1]. A height that is measured relative to a geoid is known as an orthometric height.

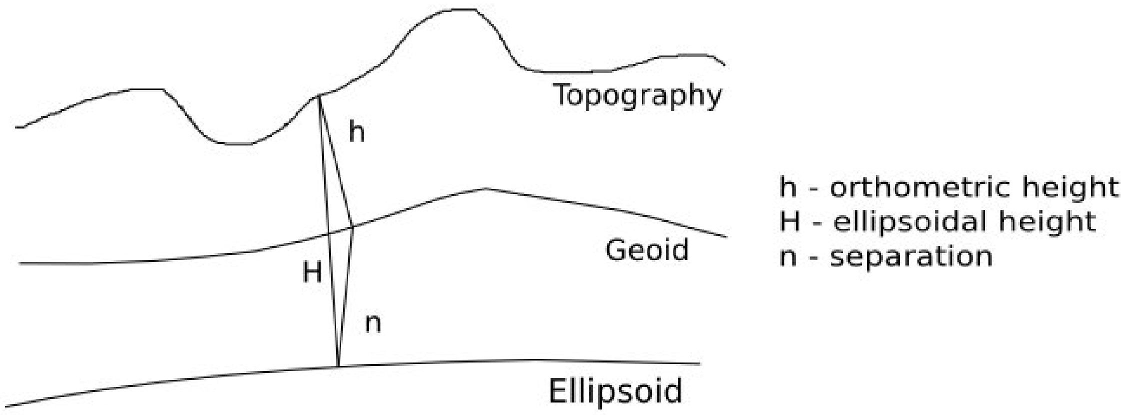
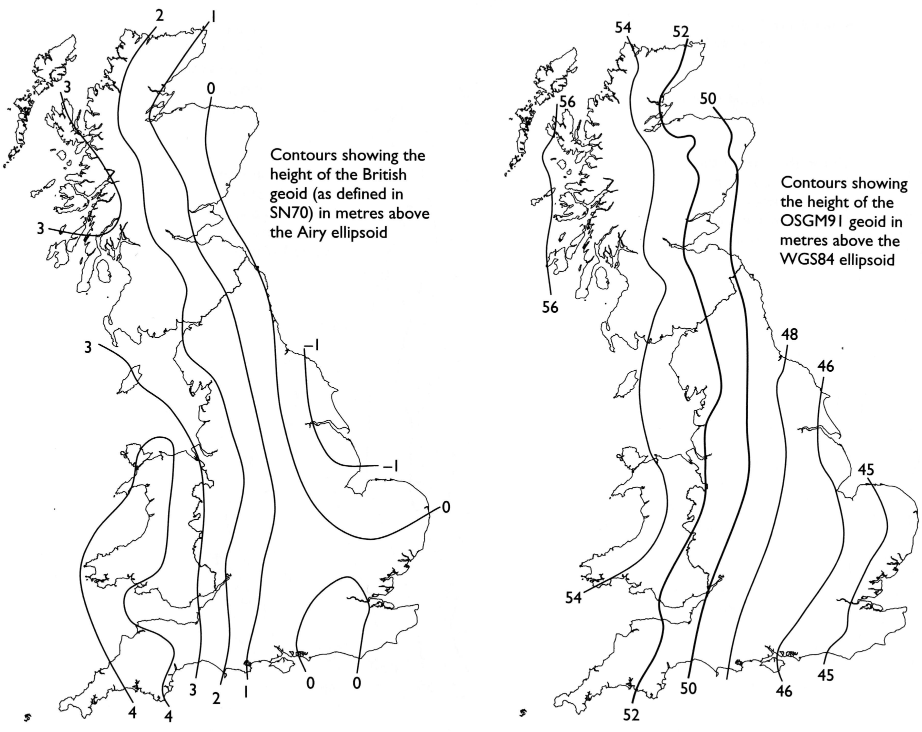


Figure 2: Diagram showing the difference between ellipsoidal and orthometric heights

Orthometric heights are much easier to measure (such as by the process described in section 3.1) as the geoid is orthogonal to the gravitational field. However as the above diagram shows if an instrument is set up level it will be aligned with the orthometric height but not necessarily the ellipsoidal height.

2.2 Ellipsoid

An ellipsoid is a mathematical representation of the surface of the Earth and is defined by its centre, major & minor radii and the direction of its principle axes. Points can then be defined in either the Cartesian coordinate system or by latitude and longitude and the height above the ellipsoid surface. Heights measured from the ellipsoid are known as ellipsoidal heights. The ellipsoid commonly used in Europe for GPS is called ETRS89(European Terrestrial Reference System 1989) and is based on the the WGS84 ellipsoid. The WGS84 ellipsoid approximates the whole earth however it is rotating so that its movement relative to all the tectonic plates is zero, the ETRS89 ellipsoid however follows the Eurasian plate so coordinates of points will remain constant. The Airy ellipsoid closely follows the British geoid and is used for OSGB36.



The British geoid on the Airy ellipsoid.

The British geoid on the ETRS89 ellipsoid.

Figure 3: Comparison of British Geoid and two commonly used ellipsoids

As can be seen from the above diagrams the range in difference between the British geoid and the ellipsoids is around 5m for the Airy ellipsoid and 11m for the ETRS89 ellipsoid, this is due to the fact the two ellipsoids are in slightly different positions. The transform introduced later on in this report places an ellipsoid directly on the geoid in Cambridge; as a result there will be very little separation between the two in the Cambridge area.

2.3 Projections

In order to convert points from a three dimensional ellipsoid to a two dimensional plane a transformation is used. In Britain a transverse Mercator projection is used however this is just one of many possible projections. The transverse Mercator is produced by placing a cylinder over the ellipsoid with its major axis in the same plane as the equator. When the cylinder is unwrapped the areas close to the centre have very little distortion but at the edges there is a large amount of distortion. This projection produces a conformal map; this means that an angle measured on a map would be the same as an angle measured with a theodolite in the field.

2.4 Transformation

To obtain Eastings and Northings in OSGB36 from ETRS89 the OSTN02 transformation is used, the OSGM02 transformation is used to obtain orthometric heights. However the OSTN02 transform is not homogeneous in that Britain is divided into 1km squares with the transformation being slightly different for each square. This means that if the boundary between two squares falls on an area that is being surveyed, there will be a step in the data and angles and distances between points will be lost when the data is transformed.

2.5 GNSS

GNSS (Global Navigation Satellite System) is a modern surveying technique which utilises a network of satellites to find the location of points on the ground. Each satellite transmits two signals, one contains the location of the satellite and the other is a time signal. Each satellite has a highly accurate atomic clock on board that are in sync with each other and the receivers on the ground. The receivers measure the time between when the satellite sends a time signal and when it is received; assuming that the signal has travelled at the speed of light, the receiver can calculate its distance from the satellite. This distance is called a pseudo range as there are a number of errors in this distance that are described below. With only one satellite the possible position of the receiver is somewhere on the surface of a sphere whose radius is the distance that has been measured. When a second satellite is used the locus of possible locations is reduced to a circle, a third reduces this to two points and a fourth to the actual location.

2.5.1 Errors in GNSS measurements

- The first source of possible errors is the clocks, both on the satellites and in the receivers. Since the speed of light is so large even a very small error with the time will cause positional error of many meters, for example a time error of 1μs will give a positional error of 300m. Each satellite has four atomic clocks on board that will gain or lose a second approximately every 32,000 years[2]. The time of each satellite is monitored by stations on the ground that can transmit messages to make adjustments if necessary. If every receiver had clocks as accurate as those on the satellites the cost would be uneconomical; instead cheaper clocks are used and the fourth satellite that is being observed is used to remove the error. If there is an error in the receiver clock it is likely that the pseudo ranges of the first three satellites will still overlap to produce two possible locations, however these points will not lie on the surface of the sphere produced by the fourth pseudo range. Adjustments are made by the receiver so that the four spheres intersect at approximately the same point.

- The second source of error is the speed at which the signal travels at. Although this is constant in the vacuum of space it cannot be assumed to be constant in the Earth's atmosphere. This is because the atmosphere is made up of layers of differing density, as light passes through these layers it is refracted so does not take a straight path. The densities are affected by many factors such as temperature, humidity and solar activity. Signals coming from satellites directly overhead have less atmosphere to pass through than those closer to the horizon so are less affected by this. A way to reduce this error is to use DGNSS as outlined in section 2.5.2.
- A final source of error is called multipath. This is when the receiver uses a signal that has been reflected off of a nearby surface such as a wall. As a result the receiver thinks it is located where the reflected signal would have been if there was no obstruction. Multipath is hard to detect and correct; ways to eliminate multipath is through repeated measurements and by using locations that have a clear view of the sky and are not located too close to buildings or beneath trees. However these are not 100% effective, combining GPS measurements with conventional observations will cause any multipath errors to become apparent.

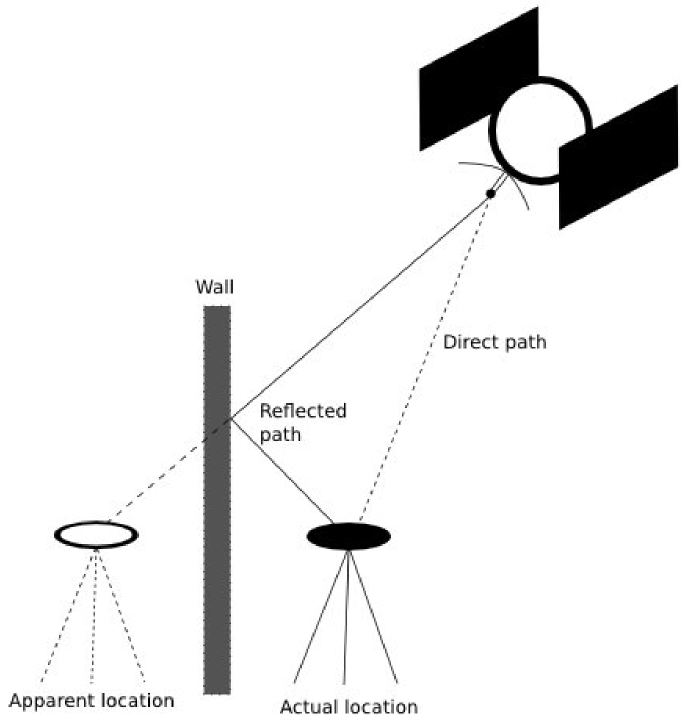


Figure 4: Diagram showing the effects of multipath

2.5.2 DGNSS

Differential Global Navigational Satellite System is used to try and reduce errors caused by the atmosphere and also if the satellites are not in the exact location that they state the are. It is done by placing a receiver at a known location and using it as a base station. The base station will be affected by the same errors as the other receivers, however, since its location is known the vector from its actual and apparent locations can be calculated. This error vector will be equal to that of the other receivers as well, so their apparent location can be adjusted to give their actual location. DGNSS is more effective if the two receivers are close together but will give good results over a few kilometres.

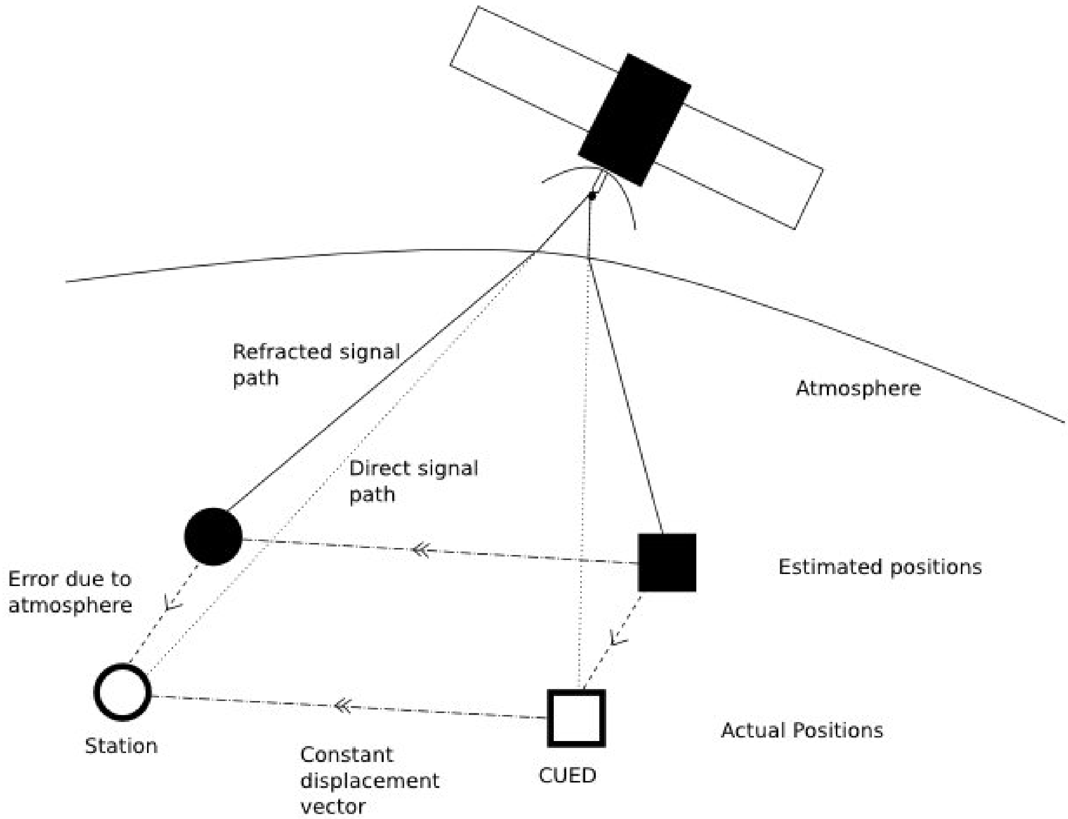


Figure 5: Diagram showing how errors can be reduced through differential GPS

2.6 Conventional Surveying

Conventional surveying is done by measuring the distance and angles between stations to find their relative positions. Simple trigonometry can be used to find the location of a new station from as little as two readings. A traverse can be performed in order to find several new stations; if two stations are known initially a third station can be found, this new station is then used to find the next station and so on. The geometry of a network of points can affect the accuracy of results. For example if there are three points in the shape of an isosceles triangle, a slightly incorrect angle will move the furthest point much more than if the points were in the shape of an equilateral triangle. An analogy can be made to pin jointed frames, the stiffer the frame, the stiffer the network of points will be. This is important with respect to the reference objects as they are far from Coe Fen and are viewed from similar directions; as a result their coordinates are the least well known.

2.7 Levelling

Levelling is the process in which the differential heights between stations are measured. A level is set up between two points and a pole with a barcode, which corresponds to the height on the pole, printed on it is placed on one of the points. The level measures the height of the instrument on the pole, this is called the backsight. This measurement is repeated with the next point and is called the foresight. As can be seen in the following diagram the difference in measured heights is equal to the difference in orthometric height of the station. These heights are all relative to each other and so the measurements need to include an Ordnance Survey benchmark, of which there are four on Coe Fen, for them to be of use. The orthometric heights of each benchmark are published and from these the orthometric heights of each station can be calculated. When the measurements are taken it is good practice to create loops by finishing on the station that was started on; if there is a significant height difference at the end then something must have gone wrong, however if the difference is small it can be assumed that the error is distributed over the loop.

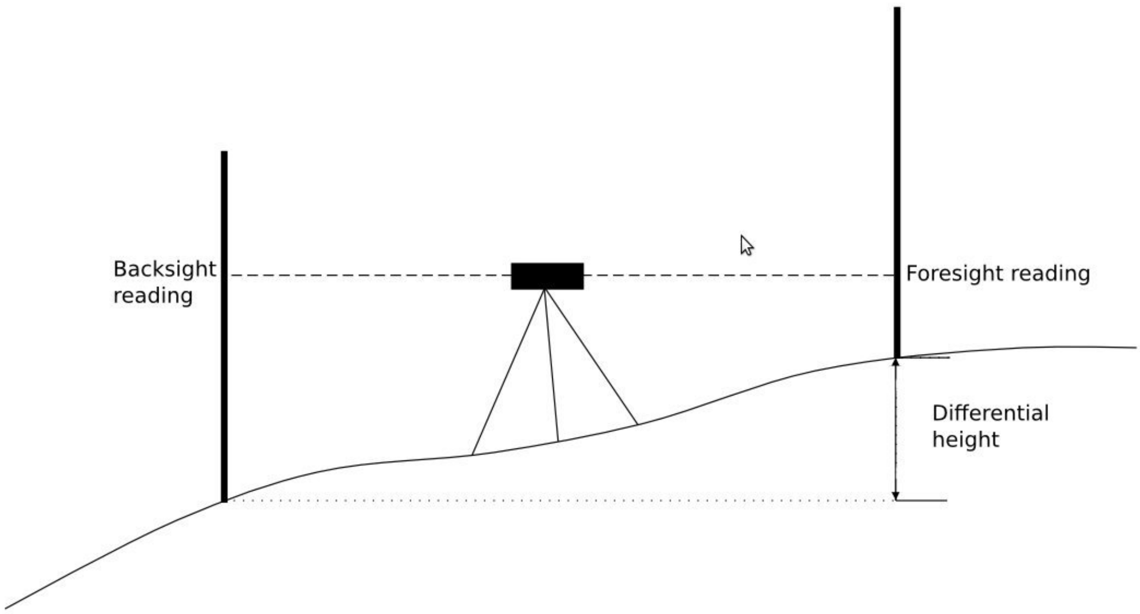


Figure 6: How a height differential between two stations can be measured

3 Apparatus and Method

In order to improve the accuracy of the station coordinates readings were taken with both GPS and conventional instruments. However, since the OSTN02 and OSGM02 transformations are not homogeneous a transform that is specific to the local area and which would not introduce distortions had to be created first.

3.1 Levelling

The first stage in producing a homogeneous transform for the Coe Fen area was to find orthometric heights for all of the stations. This was done by levelling, making sure that there were several loops, that included local ordnance survey benchmarks, to create redundancy. The DNA03 Digital level has an accuracy of 0.05mm, however the data will only be used to an accuracy of 1mm as will be shown later The heights obtained through GPS measurement were not used as they are only accurate to within a few centimetres, while the EAA requires mm accuracy. The inaccuracies arise for two reasons; the first is due to the fact that the separation between the geoid and the ellipsoid is not known exactly at every location but only at the corner of every kilometre square and the OSGM02 transform is only accurate to 20mm[3]. The second is due to the geometry of the GPS satellites, since they are all above the point that is being measured the height coordinate is the least accurate. Levelling data from previous years was processed using LSQ, a least squares program for adjusting surveying data. The LSQ program allows each measurement to include an estimated standard deviation(ESD), this essentially puts a spring between it and the other measurements. If a measurement is given a higher ESD, for example if there was heat haze that could have affected the measurement, it will not have as stiff a spring so will have less effect on the final position of points. Points can also be assigned to be fixed, adjustable or constrained. When processing the levelling data all of the stations were made adjustable and the Ordnance Survey benchmarks were given 1m constraints. However this produced different results to last year despite the same data being used. It was discovered that last year the constraints on the benchmarks was only 5mm which may have been too small and was causing the network of points to be distorted:

Table 1: Comparison of benchmark heights with different constraints

Benchmark	5mm Constraint (m)	1m Constraint (m)	Difference (m)
BMW	7.5220	7.5194	-0.0026
BMX	8.1244	8.1260	0.0016
BMY	9.6260	9.6286	0.0026
BMZ	8.1489	8.1473	-0.0016

Table 2: Comparison of station heights with different constraints

Station	5mm constraint	1m constraint	Station	5mm constraint	1m constraint
AB	6.3631	6.3636	CB	7.2664	7.2670
AC	6.0778	6.0784	CC	7.5859	7.5864
AD	5.8679	5.8685	CD	7.2429	7.2435
AE	7.7447	7.7453	CE	7.5199	7.5104
BB	6.6606	6.6612	DB	7.4322	7.4327
BC	6.4491	6.4497	DC	7.7465	7.7471
BD	6.6736	6.6742	DD	7.3391	7.3397
BE	7.0363	7.0369	DE	7.4489	7.4494

3.2 Producing the transform

The next step in the process is to collect DGNSS (Differential Global Navigation Satellite System) measurements. At this stage only data from previous years has been used. The OSTN02 and OSGM02 transforms were used to convert the ETRS89 Cartesian coordinates into OSGB36 Eastings, Northings and orthometric heights; these points are not currently homogeneous. Since we do not want to use the heights measured by DGNSS the heights that were obtained by levelling and adjusted with LSQ are over written onto all of the points, this set of points are now the target data. Leica GeoOffice software is then used to compute a transformation between the original ETRS89 coordinates to the target data. This transform places an Airy ellipsoid, already a good approximation of the geoid in Britain, that has been translated and rotated so that its surface closely follows the geoid in Cambridge. This means that the ellipsoidal heights are equal to the orthometric heights in the area so when future DGNSS measurements are processed with this transform the heights will be more accurate. This transform is also homogeneous and unlike OSTN02/OSGM02 will not cause any steps in the data. However this transform is only suitable for the Cambridge area; if it was used in another part of Britain the results would be of no use due the fact that shifts produced by OSTN02/OSGM02 are different in other areas of the country.

Table 3: Comparison of transform parameters

	Sturrock	Downes/Dempster
dx	-489.37959m	-488.81718m
dy	221.05326m	221.64892m
dz	-342.52472m	-342.96562m
rx	-2.78418”	2.74322”
ry	-4.87732”	-4.85405”
rz	2.22690”	2.3107”

The above table shows the difference between the parameters used in this years transformation and last years. The translations and rotations place different “tilts” on the ellipsoids, however both ultimately produce similar results. We can see that the translation parameters are around half a meter different, this is caused by the half a millimetre difference in heights seen in Table 2.

3.3 GPS Measurements

In order to make differential GPS measurements a base station had previously been set up on the roof of the Engineering Department. A Leica GPS System 500 receiver was attached to a bracket on the roof and was left to run for several hours to find its approximate location. Ordnance Survey run a network of around 50 active GPS stations whose coordinates are known very precisely. These stations are constantly measuring their position and the data that they collect is published online in RINEX (Receiver INdependent EXchange) format[4]. The RINEX data from five nearby stations for the time when the receiver was running on the CUED roof was downloaded. This data was entered into Leica GeoOffice software along with the measurements taken on the roof and a least squares calculation was performed so that the coordinates of the bracket are now known to an accuracy of around 1mm. This station (CUED) could now be used as a base station for all future readings.

When measuring stations on Coe Fen one receiver is set up at point CUED (base station) and one receiver is set up at the desired station(rover). The rover is mounted to a levelled tripod that is directly above the station, the height between the station and the antenna is measured and is entered into the controller. The receiver is left running for around 30 minutes, taking a reading every 15 seconds. This time period is sufficient to resolve the initial phase ambiguity, the number of phases between the satellite and receiver, which increases the accuracy from around 1m to 20mm. The readings from the rover and base station are processed with GeoOffice to remove any errors introduced by atmospheric effects.

This process was done at stations AB, AC, AD, AE, BB, CC, CD, CE and DC to supplement the data that already existed and also on the roof of the University Library tower and Queens' College as described in section 3.4.1 to improve the distant reference objects.

3.4 Conventional Measurements

A Kern DKM3 was used for all angle measurement between stations. This instrument can measure angles to an accuracy of 0.1"±.5", which equates to approximately 0.5mm over a distance of 1km. In order to improve the coordinates of a station the theodolite was placed directly over the station and targets were placed on other stations in the area. Horizontal angles were measured between the stations and at least one reference object.

Table 4: Measurements made with the DKM3

Station Measurements taken from	Observed
AB	AC, RCSP
AE	AC, AD, KCSW
BB	BC, BD, CHIM
CD	CC, CE, CHIM
CE	CC, CD, CHIM, RCSP
DB	DC, DD, CHIM
DE	DC, DD, CHIM, KCSW
UL	CHIM, KCSW, RCSP

3.4.1 Additional points

The coordinates of the Roman Catholic Church (RCSP) are well known as it is a 3rd order Ordnance Survey point and the Engineering Department chimney (CHIM) is viewed by several stations so there is a higher confidence in its coordinates. However Kings College tower (KCSW) is only viewed by a few stations and they are all from a similar direction and as a result it has quite a large error ellipse. It was decided that in order to reduce this ellipse a new station was to be created. Using online mapping sites several locations were found where all three reference objects could be viewed from. These included the roof of the University Library tower (UL) and a balcony in Cripps Court, Queens' College (QCCC). The library tower used to have an old Ordnance Survey point however this has since had an aerial placed over it.

In order to create the new stations a temporary point was made and GPS receiver was set up and left to run for around 1 hour. The DKM3 was then set up on the point and measurements were taken. Since three angles were measured and the distances between the reference objects was known it was possible to calculate the coordinates of the new location. However if the coordinates were measured with GPS first, the readings made with the DKM3 would put further constraints on the reference objects and increase our confidence in their coordinates.

4 Results and Analysis

4.1 Levelling

Since this project utilises levelling data collected by Downes and Dempster in 2010/11 the first thing to do was to check the data. The readings were checked to ensure that there were a sufficient number of loops and measurements taken to create redundancy. The following figure shows the network of levelling data:

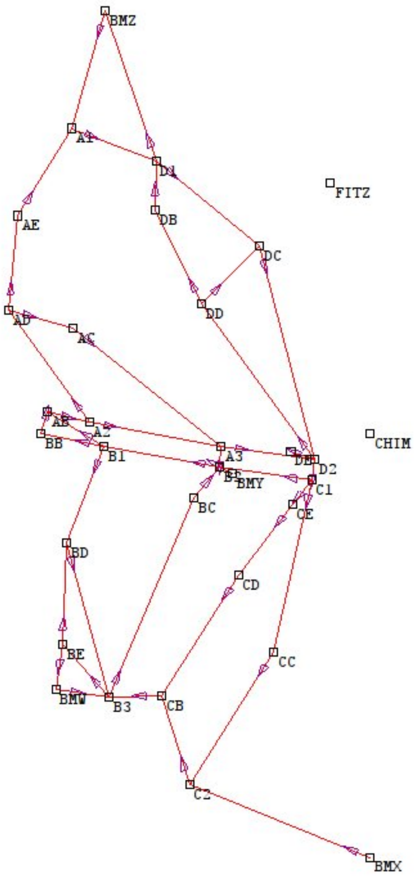


Figure 7: Levelling network

As discussed in section 3.1, the benchmarks had previously been constrained to 5mm. However, when the constraint was relaxed there was further movement in the system indicating the 5mm constraints may have caused it to become distorted slightly. It should also be noted that one of the benchmarks, C2, is located on a bridge that has been dismantled and rebuilt since it was an Ordnance Survey benchmark so is unlikely to still be at the published height.

When the data was processed with LSQ the maximum weighted difference was 0.39, which indicates that there are no measurements that are fighting with each other. However the ESD scale factor was 0.22, which is quite low. A scale factor of 1 suggests that each measurement has been given a good ESD, this low value suggests that the ESDs are too large for the data. Unfortunately LSQ is unable to accept an ESD of less than 1mm for height measurements and the data supplied is more accurate than this. The following plot compares the distribution of weighted errors with a normal distribution curve:

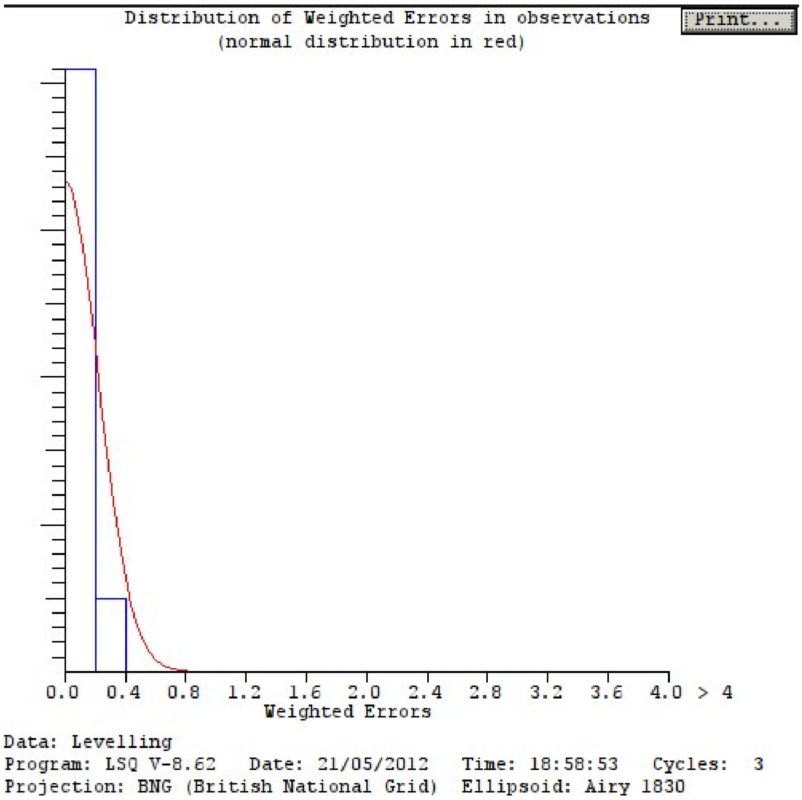
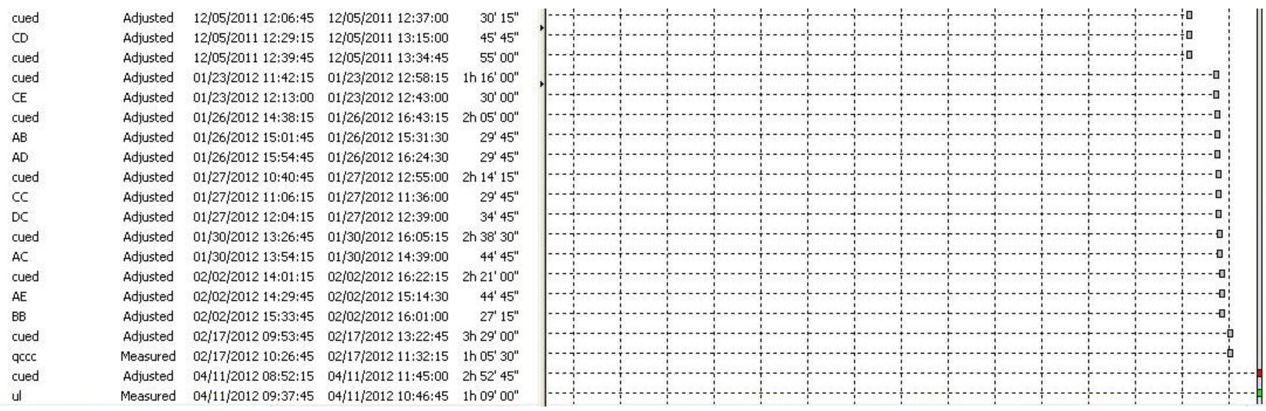


Figure 8: Distribution of levelling errors

4.2 GPS

The GPS data that was collected was processed with Leica GeoOffice software. This software did the differential calculation to remove the errors introduced due to atmospheric effects as shown in the figure below. It also combined the data collected this year with the data from previous years to produce an averaged set of coordinates by a least squares calculation.



cued	Adjusted	12/05/2011 12:06:45	12/05/2011 12:37:00	30' 15"
CD	Adjusted	12/05/2011 12:29:15	12/05/2011 13:15:00	45' 45"
cued	Adjusted	12/05/2011 12:39:45	12/05/2011 13:34:45	55' 00"
cued	Adjusted	01/23/2012 11:42:15	01/23/2012 12:58:15	1h 16' 00"
CE	Adjusted	01/23/2012 12:13:00	01/23/2012 12:43:00	30' 00"
cued	Adjusted	01/26/2012 14:38:15	01/26/2012 16:43:15	2h 05' 00"
AB	Adjusted	01/26/2012 15:01:45	01/26/2012 15:31:30	29' 45"
AD	Adjusted	01/26/2012 15:54:45	01/26/2012 16:24:30	29' 45"
cued	Adjusted	01/27/2012 10:40:45	01/27/2012 12:55:00	2h 14' 15"
CC	Adjusted	01/27/2012 11:06:15	01/27/2012 11:36:00	29' 45"
DC	Adjusted	01/27/2012 12:04:15	01/27/2012 12:39:00	34' 45"
cued	Adjusted	01/30/2012 13:26:45	01/30/2012 16:05:15	2h 38' 30"
AC	Adjusted	01/30/2012 13:54:15	01/30/2012 14:39:00	44' 45"
cued	Adjusted	02/02/2012 14:01:15	02/02/2012 16:22:15	2h 21' 00"
AE	Adjusted	02/02/2012 14:29:45	02/02/2012 15:14:30	44' 45"
BB	Adjusted	02/02/2012 15:33:45	02/02/2012 16:01:00	27' 15"
cued	Adjusted	02/17/2012 09:53:45	02/17/2012 13:22:45	3h 29' 00"
qccc	Measured	02/17/2012 10:26:45	02/17/2012 11:32:15	1h 05' 30"
cued	Adjusted	04/11/2012 08:52:15	04/11/2012 11:45:00	2h 52' 45"
ul	Measured	04/11/2012 09:37:45	04/11/2012 10:46:45	1h 09' 00"

Figure 9: Processing of data with Leica GeoOffice

A way to analyse the quality of the GPS measurements is too look at the standard deviation in the readings. The table below shows the mean standard deviation for each point's Easting, Northing and height coordinate:

Table 5: Standard Deviations of GPS Data

Standard Deviations	Mean (mm)
Easting	3.02
Northing	4.23
Height	7.91

As we can see the mean height standard deviation is around double the other standard deviations. This was to be expected due to the geometry of the satellites in the sky as has been discussed before. This is also one of the reasons why the GPS heights have been disregarded and levelled heights have been used in their place. Station CUED has Easting and Northing standard deviations of .83mm and 1.16mm respectively. These values are so low for several reasons; the first is that the coordinates of this point were calculated were calculated using the RINEX data from the active stations which are known to a high degree of accuracy, the second reason is that the point is located in an elevated position and has unobstructed views of the sky so is unlikely to suffer from multipath and the geometry of the satellites will be better than if it was on the ground. The lowest standard deviations are 0.24mm and 0.32mm, these belong to the station UL. This is the highest station and there are no objects nearby for the signal to reflect off of. Measurement have only been made from this station for around one hour so it has far fewer readings than CUED which may be another reason as to why it has low standard deviations. A way of quantifying the geometry of the satellites is by the GDOP (Geometric Dilution of Precision); this takes into account the relative positions of all satellites and the receiver, a GDOP is the ideal value. Included in appendix C is a plot of the GDOP against time for the UL station, the mean value was 3.017 which is very good. The figure below shows a plot of all the stations, their associated error ellipses and the differential vectors, the six vector extending off of the page are from the active stations. Station BE has the largest ellipse, this may be due to the fact that is located in an area with quite a few trees which may have caused some of the measurements to have been affected by multipath. Station BB on the other hand is in relatively open land and has a smaller ellipse

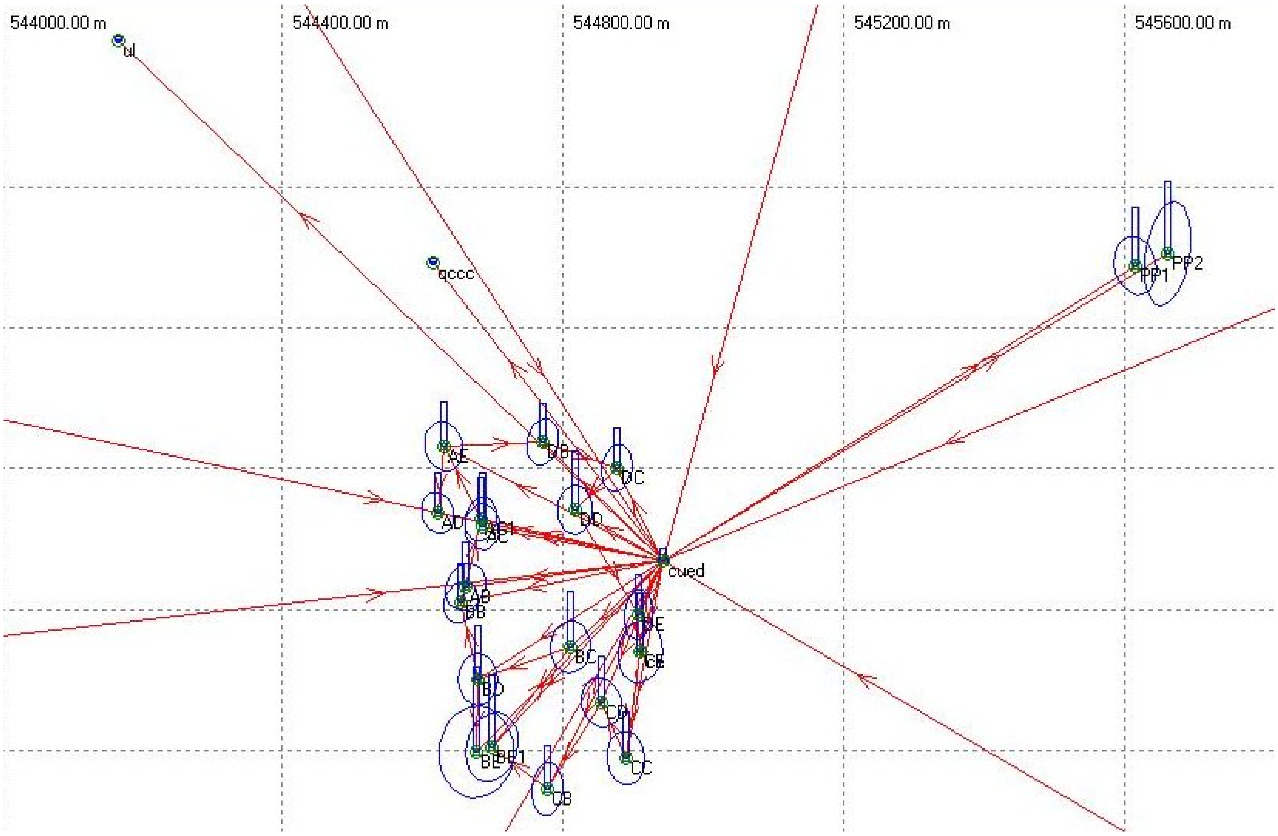


Figure 10: Ellipse and differential vectors of GPS measurements

4.3 Conventional Measurements

In total 28 sets of readings were taken however many of these were not used (The final set of readings taken can be found in appendix D and the full set of readings can be found in the accompanying log book). Some of the measurements that were made caused conflicts to arise and as a result large ESDs of 10 seconds had to be placed on problem readings in order for them to agree with the existing data. Considering the high accuracy that the instrument can achieve, these measurements were not of much use so were repeated until a useful set was obtained. The errors in the measurements may have been due to not setting up the instrument/target perfectly or human error in the reading of the scale. Measurements from QCCC had to be disregarded for a quite different reason however; the instrument was placed on a wooden decking which unfortunately flexed under the movement of the user, meaning that it was no longer level and the measured angels were different to the actual values. The measurement of the lightning rod on the chimney from point CE still has an ESD of 5 attached to it. The point CHIM is defined as the lowest point on the lightning rod. However it is at a slight angle and the lowest visible point depends on where it is being viewed from. CE is on the opposite side of the chimney from the lightning rod and if the DKM3 is set up looking at where the rod should end, the crosshairs are at a point on the brick work where the line of the lightning rod may extend to.

4.4 Transform

The following plot shows the residual errors produced by the new transform. These are the differences between the coordinates that have been transformed with OSTN02 and the coordinates processed by this projects transform:

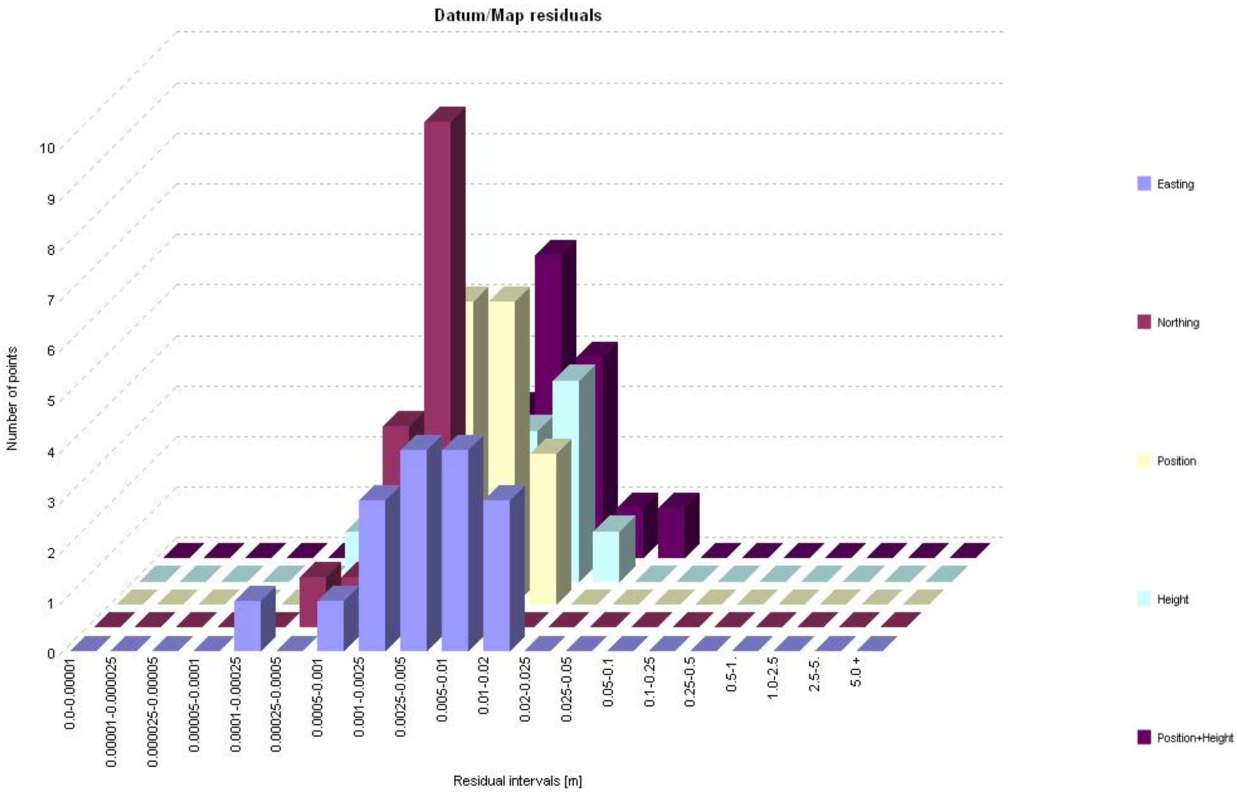


Figure 11: Residual Errors from Transform

As we can see the residuals are approximately normally distributed with a mean difference of around 2.5-5mm, suggesting the residuals are spread throughout the points. There are no residuals of 0mm as this transform was produced by a least squares calculation and would mean that high residuals would have to exist. There are no residuals greater than 20mm which is good as this is within the accuracy of GPS.

4.5 Final Coordinates

To calculate the final coordinates the GPS coordinates were entered into LSQ as fixed points. Attached to these anchors were a set of loose points that had an estimated standard deviation of 5mm. The conventional measurements were applied to these loose points so that the agreement between the two methods could be seen. If the conventional measurements put a lot of strain on one of the points it suggests that the

GPS data for that point may need to be checked.

When the full set of data was adjusted with LSQ, the distribution of weighted errors was approximately normally distributed as shown in the following plot. This was as expected and show that appropriate ESDs were used, with the final scale factor being 0.955. The disproportionally high number of weighted errors in the 0-0.2 range is due to the inclusion of levelling data which had very low weighted errors as previously discussed.

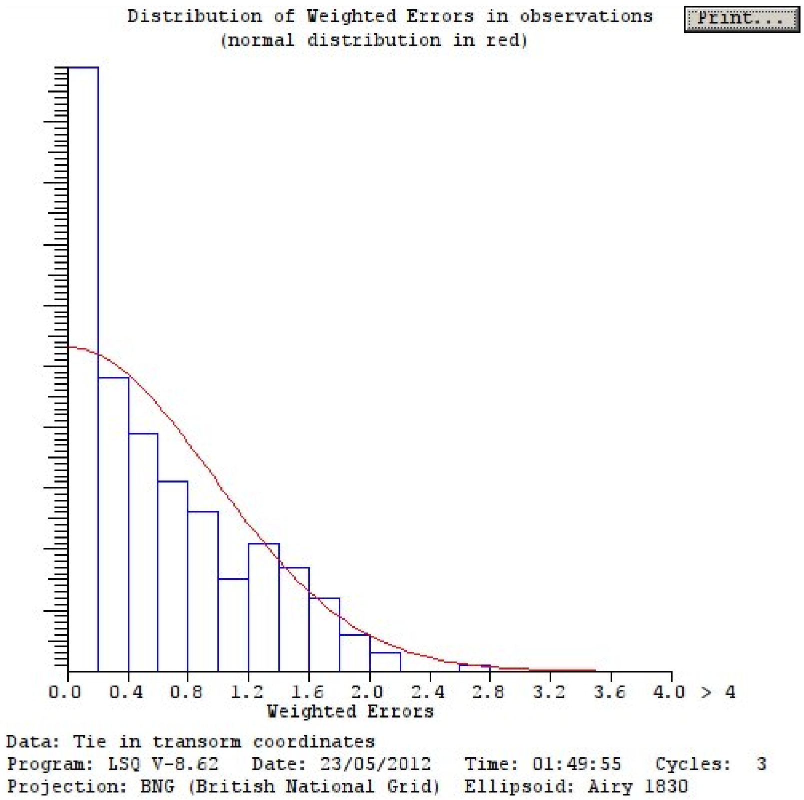


Figure 12: Error Distribution For All Measurements

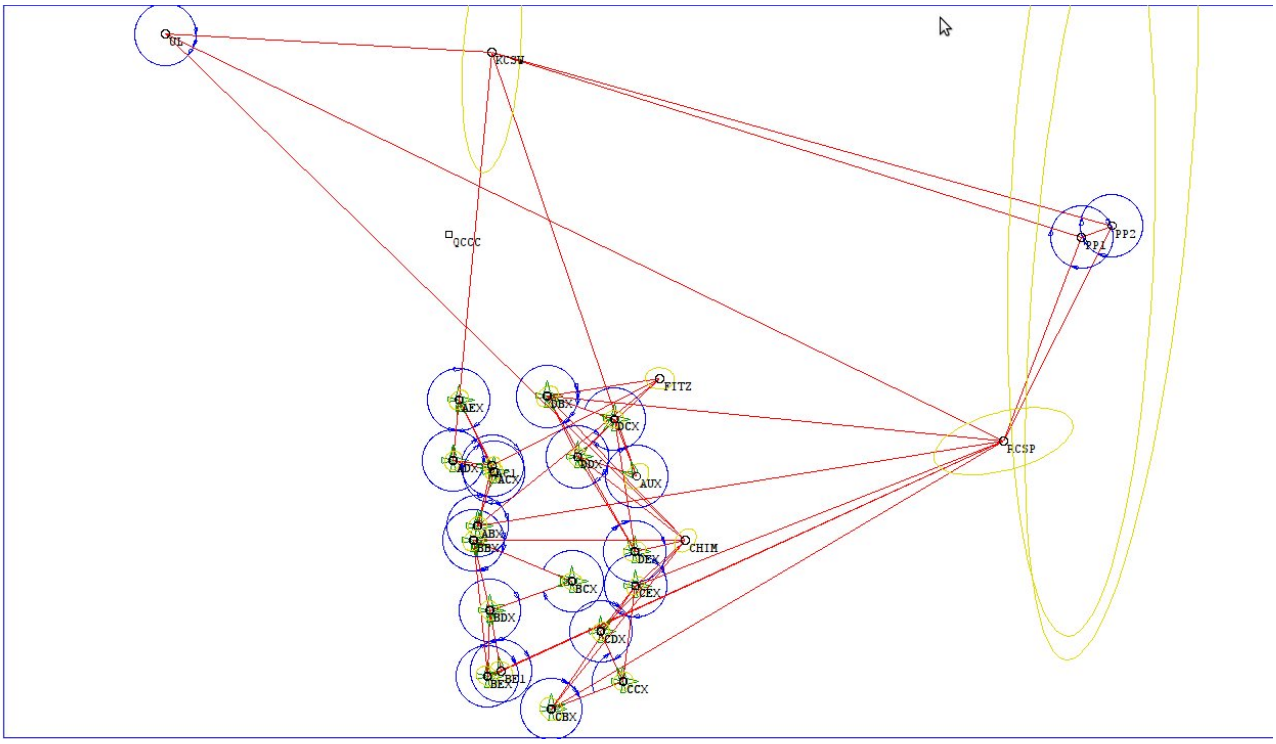


Figure 13: Error Ellipses

The above figure shows all the stations and their error ellipses as calculated by LSQ. KCSW and RCSP have the largest ellipses due to their distance from Coe Fen, however as the following table shows the area of KCSW has decreased by 65.7%; this is mainly due to the point UL which is located in an optimal position to reduce the major axis radius of the ellipse. The ellipses of the other two reference objects, RCSP and CHIM have been reduced by 29 and 25.1% respectively. The average decrease in ellipse area is 16.1% and the stations in area D had the greatest decrease in area. The radius of each ellipse is equal to one standard deviation and so we can say with 60% confidence that the point lies within this ellipse[6].

When we look at how far the conventional measurements try to shift the GPS points we see that the point that is pulled the most is CC at 10.9mm, still within the expected error of GPS measurements. This point is situated close to a tall tree line so has a large proportion of the sky blocked and may also have suffered slightly due to multipath. The error ellipses of both the GPS data and the conventional data are not that large, this tells us that the measurements made this year support previous measurements but the discrepancy between methods has not been fully determined. Station CE is only pulled 0.6mm away from its GPS location showing very good agreement between the GPS and conventional measurements.

Table 6: Change in Ellipse Area

Station	Area Before (mm^2)	Area After (mm^2)	% Change
AB	19.604	16.588	-15.4
AC	15.834	14.451	-8.7
AD	16.493	15.080	-8.6
AE	18.473	16.965	-8.2
BB	22.619	20.232	-10.6
BC	17.279	15.080	-12.7
BD	19.604	18.064	-7.9
BE	20.735	20.954	1.1
CB	25.955	24.347	-9.7
CC	18.787	16.493	-12.2
CD	17.153	14.326	-16.5
CE	16.965	14.703	-13.3
DB	25.290	20.735	-18.0
DC	18.787	16.588	-11.7
DD	16.493	14.451	-12.4
DE	24.567	20.043	-18.4
RCSP	578.304	401.338	-30.6
CHIM	31.856	23.122	-27.4
KCSW	1949.233	669.159	-65.7

5 Conclusions

We have seen that when using instruments that are capable of high accuracy measurements it is very easy to introduce errors through human error and that repeated measurements are key to ensuring the results are accurate. While the instruments used in the EAA are not capable of reading angles to 0.1" its is important that we can be confident the accuracy of the coordinates falls within that of the instrument being used.

While GPS can be an convenient way to find coordinates it is very easy for errors to be introduced and they can easily go unnoticed. If accurate heights are required it may be more beneficial to use another technique such as levelling since the accuracy of heights is approximately half that of position. In order to obtain precise positions it is advantageous to use differential GPS, however this requires a base station with reliable coordinates. One way to improve GPS measurements is by creating a transform specific to the area that is to be surveyed. This will remove the need for the non-homogeneous OSTN02 and OSGM02 transforms and may also improve the height accuracy as there is no need for a geoid model.

Traditional surveying techniques can produce results that agree well with GPS surveying and can be useful in checking if any systematic errors have occurred in GPS measurements, although it is a more time consuming process and requires much more effort.

While the error ellipse for KCSW has been greatly reduced in size it is still quite large. Future work could be done to reduce this by the creation of another point outwith Coe Fen. One possible location is on the top floor of the multistory car park in the centre of town, which may help reduce the RCSP ellipse as well. There is also the possibility that sight lines will change in the future due to the growth of vegetation or construction. During this project trees were cut down in the Leys School and RCSP is now visible from station CE. If a station does need to be moved in the future the transform from this project will allow the new coordinates to be found more accurately than if OSTN02 is used.

References

[1] Bing Maps, online, www.bing.com/maps, accessed 15/5/12

[2] Wikipedia, Ordnance Datum, online, http://en.wikipedia.org/wiki/Ordnance_Datum, accessed 14/5/12

[3] NASA, GPS Atomic Clocks, online, http://science.nasa.gov/science-news/science-at-nasa/2002/08apr_atomicclock/, accessed, 16/5/12

[4] Ordnance Survey, OSNet, online, http://www.ordnancesurvey.co.uk/oswebsite/gps/osn accessed 18/5/12

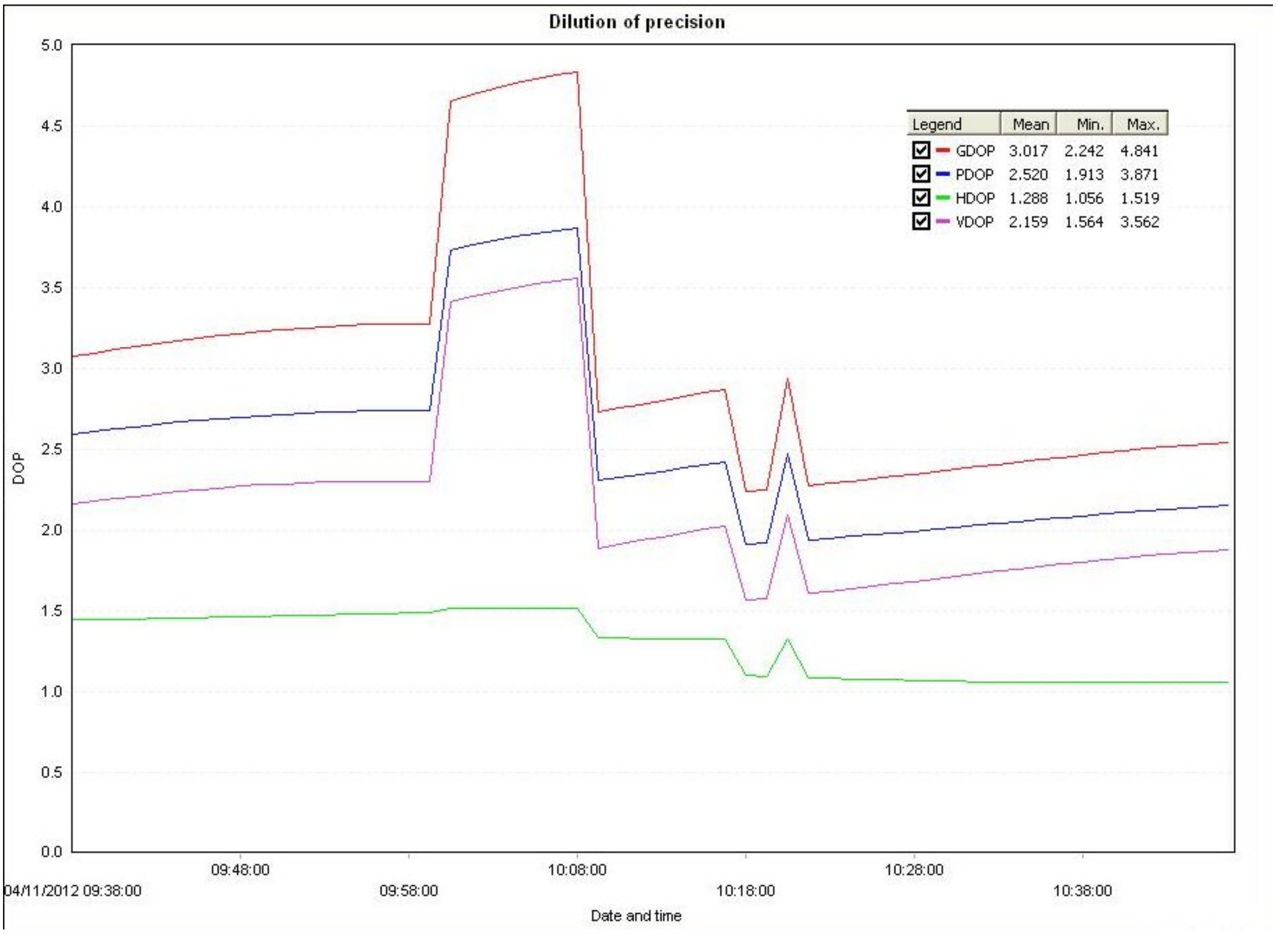
[5] Ordnance Survey, OSNet, online, http://www.ordnancesurvey.co.uk/oswebsite/gps/osn accessed 19/5/12

[6] Johnson, Aylmer, LSQ Manual, Guidance on Normal Use

A Risk Assessment

Before any work was done on the project a risk assessment was completed to identify any possible sources of risk. The greatest source of risk was thought to be working on rooftops. The risk was minimised by staying away from the edge, on the correct side of the railing and by being accompanied by a member of staff while on top of the University Library tower. Appropriate clothing was worn while collecting measurements; boots were worn and warm clothes during the winter months.

B GDOP plot for station UL



C Map of Coe Fen

D LSQ File


CUED SURVEY STATIONS
8 March2011
Each grid square has 100 m sides
Stations are plotted to the limits of printing accuracy; other detail is approximate, and is intended only as a guide
Peterhouse
Trumpington Street
Garden House Hotel
Fitzwilliam Museum
FITZ
Engineering Laboratories
CHIM
Fen Causeway
Leys School
BM_Z
AL
D1
DB
AE
DD
AD
AC
AB
AA
BB
B1
AJ
B2
BM_Y
DE
D2
C1
CE
BC
BD
CD
BE
BM_W
B3
CB
CC
C2
BM_X
258100
258000
257900
257800
257700
257600
257500
257400
257300
257200
257100
257000
256900
544500
544600
544700
544800
544900
545000
545100
545200
545300

#STATIONS

* Distant reference objects
RCSP 545490.8521 A 257766.7190 A 60.0000 F
FITZ 544948.7439 A 257865.7330 A 50.0000 F
CHIM 544989.1459 A 257610.3445 A 50.0000 F
KCSW 544683.6975 A 258380.2516 A 60.0000 F

* Ground stations
AB 544661.43387 A 257632.76361 A 6.3632 A
AC 544687.21376 A 257718.24163 A 6.0780 A
AD 544622.57580 A 257736.52506 A 5.8681 A
AE 544631.68217 A 257832.16496 A 5.7449 A

BB 544655.00502 A 257610.46842 A 6.6608 A
BC 544810.42468 A 257545.72612 A 6.4493 A
BD 544680.69418 A 257499.89997 A 6.6737 A
BE 544676.73107 A 257395.89866 A 7.0365 A

CB 544777.49524 A 257343.99460 A 7.2666 A
CC 544891.03922 A 257388.08135 A 7.5860 A
CD 544855.52903 A 257466.68793 A 7.2431 A
CE 544910.51153 A 257538.61622 A 7.5200 A

DB 544771.22053 A 257837.70158 A 7.4323 A
DC 544876.80696 A 257801.64781 A 7.7466 A
DD 544818.83244 A 257742.08781 A 7.3393 A
DE 544909.03474 A 257592.92346 A 7.4490 A

* Auxiliary points for additional conventional observations
AUX 544912.1358 A 257711.5500 A 7.0000 F * to fix KCSW from the fen
PP1 545614.0941 A 258088.8694 A 11.9986 F
PP2 545661.6926 A 258106.7154 A 12.0713 F
AC1 544684.2468 A 257728.6580 A 5.9125 F
BE1 544698.5633 A 257404.3159 A 6.7247 F
QCCC 544616.7934 F 258092.9524 F 19.2531 F
UL 544168.86091 0.05 258409.20757 0.05 66.88631 F

#ANCHORS

ABX 544661.43702 F 257632.76199 F 6.3632 F
ACX 544687.21638 F 257718.23975 F 6.0780 F
ADX 544622.58037 F 257736.52213 F 5.8681 F
AEX 544631.68670 F 257832.16146 F 5.7449 F

BBX 544655.00830 F 257610.46686 F 6.6608 F
BCX 544810.42325 F 257545.72719 F 6.4493 F
BDX 544680.69644 F 257499.89957 F 6.6737 F
BEX 544676.73319 F 257395.89895 F 7.0365 F

CBX 544777.49426 F 257343.99662 F 7.2666 F
CCX 544891.03500 F 257388.08458 F 7.5860 F

CDX 544855.52605 F 257466.69015 F 7.2431 F
CEX 544910.50709 F 257538.61871 F 7.5200 F

DBX 544771.22091 F 257837.70003 F 7.4323 F
DCX 544876.80414 F 257801.64803 F 7.7466 F
DDX 544818.83120 F 257742.08762 F 7.3393 F
DEX 544909.03048 F 257592.92556 F 7.4490 F

#BENCHMARKS

* OS Benchmarks
*Constraints are small (stiff springs) to allow for small movements over time since last maintenance
BMY 544850.0000 F 257570.0000 F 9.6286 1
BMZ 544720.0000 F 258040.0000 F 8.1473 1
BMX 544990.0000 F 257180.0000 F 8.1260 1
*C2 544806.0000 F 257254.0000 F 7.6810 1 * Bridge rebuilt since this measurement was made, demoted to CUED benchmark
BMW 544670.0000 F 257350.0000 F 7.5194 1

* CUED Benchmarks (approximate Eastings and Northings)
A1 544686.0000 F 257921.0000 F 6.4585 A
A2 544704.0000 F 257622.0000 F 6.5349 A
A3 544838.0000 F 257598.0000 F 6.3840 A
B1 544719.0000 F 257598.0000 F 6.5121 A
B2 544837.0000 F 257577.0000 F 6.4672 A
B3 544724.0000 F 257343.0000 F 7.9616 A
C1 544930.0000 F 257563.0000 F 7.5579 A
C2 544806.0000 F 257254.0000 F 7.6869 A
D1 544772.0000 F 257888.0000 F 7.0484 A
D2 544933.0000 F 257584.0000 F 8.0267 A

#Tie In
DE AB ABX 0 .005
DE AC ACX 0 .005
DE AD ADX 0 .005
DE AE AEX 0 .005
DE BB BBX 0 .005
DE BC BCX 0 .005
DE BD BDX 0 .005
DE BE BEX 0 .005
DE CB CBX 0 .005
DE CC CCX 0 .005
DE CD CDX 0 .005
DE CE CEX 0 .005
DE DB DBX 0 .005
DE DC DCX 0 .005
DE DD DDX 0 .005
DE DE DEX 0 .005

DN AB ABX 0 .005
DN AC ACX 0 .005

DN AD ADX 0 .005
DN AE AEX 0 .005
DN BB BBX 0 .005
DN BC BCX 0 .005
DN BD BDX 0 .005
DN BE BEX 0 .005
DN CB CBX 0 .005
DN CC CCX 0 .005
DN CD CDX 0 .005
DN CE CEX 0 .005
DN DB DBX 0 .005
DN DC DCX 0 .005
DN DD DDX 0 .005
DN DE DEX 0 .005

CONVENTIONAL DATA

DKM3 HA's

#(Sturrock, 2012)

HO BB
BD   00 03 3.5   180 01 12.2   1
CHIM 283 08 55.9   103 07 2.7   1
BD   00 03 2.6   180 01 10.4   1

HO DE
DD   00 03 31.5   180 00 53.1   1
DC   22 26 47.5   202 24 6.9   1
CHIM 108 57 29.1   288 54 50   1
DD   00 03 32.0   180 00 52.5   1

HO CE
CC   00 03 40.4   180 02 27.8   1
CD   30 05 32.0   210 04 18.9   1
CHIM 220 19 43.6   40 18 41.1   5
RCSP 241 14 12.5   61 12 57.5   1
CC   00 03 40.9   180 02 27.0   1

HO AB
AC   00 00 46.7   180 04 23.5   1
RCSP 64 03 14.1   244 06 50.1   1
AC   00 00 46.0   180 04 23.9   1

HO AE
KCSW00 01 29.6   180 04 8.0   1
AC   148 36 49.5   328 39 28.2   1
AD   180 02 34.0   00 05 12.0   1
KCSW00 01 30.0   180 04 7.6   1

HO CD
CHIM 00 00 47.2   180 03 48.3   1

CC 112 46 11.6 292 49 16.6 1
CE 354 28 52.4 174 31 56.1 1
CHIM 00 00 47.6 180 03 48.5 1

HO UL *23
KCSW0 01 9.5 180 03 11.6 1
RCSP 22 42 41.2 202 44 44.9 1
CHIM 41 01 55.2 221 03 59.2 1
KCSW0 01 9.6 180 03 10.9 1

HO BB *7
CHIM 00 00 52.5 180 00 45.1 1
BC 22 36 32.6 202 36 21.4 1
*BD 76 50 2.0 256 54 45.4 1 further measurment has been made
CHIM 00 00 51.8 180 00 45.7 1

HO DB *4
CHIM 00 01 57.0 180 02 5.0 1
DD 17 20 58.6 197 21 14.9 1
DC 332 40 20.6 152 40 32.1 1
CHIM 00 01 56.5 180 02 3.8 1
#(Dempster/Downes, 2011)
HO PP2
RCSP 00 01 01.5 180 01 05.3 1
PP1 42 47 08.1 222 47 15.6 1
KCSW 78 57 51.1 258 57 57.7 1
RCSP 00 01 01.9 180 01 05.0 1

HO PP1
RCSP 00 00 40.7 180 00 40.4 1
KCSW 86 27 54.0 266 28 00.7 1
PP2 228 31 25.6 48 31 28.5 1
RCSP 00 00 40.3 180 00 41.2 1

HO AC
AB 00 34 38.0 180 35 53.2 1
AD 89 34 44.0 269 36 05.7 1
AE 137 48 08.2 317 49 27.6 1
AB 00 34 35.8 180 35 52.5 1

HO BE
RCSP 00 07 30.6 180 07 32.9 1
BE1 03 31 28.4 183 31 35.0 1
BD 296 47 38.2 116 47 43.1 1
RCSP 00 07 32.7 180 07 31.5 1

HO AB
RCSP 00 01 35.5 180 01 38.6 1
AC 295 59 06.9 115 59 14.1 3 * Fights with other data
RCSP 00 01 29.2 180 01 38.3 1

HO BB

CHIM 00 01 32.3 180 01 32.4 1
BC 22 37 11.4 202 37 11.3 1
BD 76 55 29.8 256 55 32.0 10 *Fights with other stuff!
BE 84 13 25.6 264 13 25.6 1
CHIM 00 01 29.3 180 01 33.2 1

HO AE
KCSW 00 00 48.7 180 00 53.1 1
AC 148 36 08.7 328 36 15.5 1
AD 180 01 55.3 00 01 56.9 1
KCSW 00 00 51.3 180 00 53.8 1

#(Mottram, 27/3/09 - 03/4/09)
HO DB
DE 00 03 38.9 180 18 34.0 1
FITZ 290 28 02.0 110 42 57.2 1
RCSP 305 04 21.1 125 19 16.1 3 * clashes with other readings...
CHIM 345 39 15.7 165 54 10.7 1
DE 00 03 38.9 180 18 34.3 1

HO CB
CE 00 12 39.1 180 06 21.3 1
CHIM 04 19 57.3 184 13 39.5 1
RCSP 25 12 37.6 205 06 18.1 1
CE 00 12 39.1 180 06 21.3 1

HO AB
AC1 00 02 53.4 180 09 12.5 1
FITZ 37 37 58.7 217 44 04.9 1
RCSP 67 29 47.6 247 35 50.0 2 * heat haze
AC1 00 02 53.8 180 09 11.9 1

HO AB
AC1 00 02 27.1 180 07 33.0 1
FITZ 37 37 31.2 217 42 30.1 1
RCSP 67 29 21.3 247 34 16.7 2 * heat haze
AC1 00 02 26.8 180 07 33.2 1

HO DE
DB 00 02 58.3 180 06 29.2 1
CHIM 107 09 45.9 287 13 20.1 4 * heat haze
DB 00 02 57.7 180 06 29.2 1

HO AE
AC1 00 02 16.3 180 04 37.3 1
KCSW 212 23 00.6 32 25 15.9 1 *No redundancy...
AC1 00 02 16.3 180 04 37.1 1

HO AC1
AE 00 07 23.9 180 03 40.5 1
FITZ 89 39 08.4 269 35 18.2 2 * bar behind, hard to see
AE 00 07 23.4 180 03 40.5 1

```
#(Jacobs/Sbiti, 2007-08)
*NB the coordinates for CD are noticably different between 2007/8 and 2008/9.
*Some observations below have therefore been omitted due to lack of
*confidence in the data and poor fit with other data
*HO CE
*CD      00 00 16.2     180 01 28.5     1
*CHIM    200 37 27.3     20 38 42.4     1
*CD      00 00 17.4     180 01 25.8     1

HO CB
*CD       0 01 22.4     180 08 02.2     1
CHIM     01 26 10.0     181 32 42.7     3
RCSP     22 18 47.2     202 25 31.4     1
*CD      00 01 24.7     180 08 01.2     1

HO BE1
BD       00 00 54.2     180 01 42.1     1
RCSP     76 01 26.1     256 02 17.4     1
BD       00 00 53.8     180 01 41.7     1

HO BB
BD       00 00 10.0     180 20 14.9     1
CHIM    283 05 53.2     103 26 01.4     5 *Conflicts with HD BB to BC
BD       00 00 11.2     180 20 12.8     1

HO AB
AC1      00 01 22.0     180 28 05.6     20 *Fights with other data
FITZ     37 36 09.1     218 02 48.7     1
RCSP     67 27 55.3     247 54 32.6     1
AC1      00 01 22.6     180 28 02.8     20

HO DB
DE       00 12 51.2     178 25 33.2     1
FITZ    290 37 15.0     108 49 55.6     1
RCSP    305 13 39.4     123 26 23.7     1
CHIM    345 48 37.3     164 01 07.8     1
DE       00 12 53.9     178 25 33.3     1

HO DD
DE       00 01 22.4     180 01 42.5     1
FITZ    257 36 08.0      77 36 31.4     1
CHIM    338 54 34.6     158 54 57.4     3
DE       00 01 22.4     180 01 42.0     1


# TC405 HD'S

#(Dempster/Downes, 2011)
HD AC
AB     89.286     0.003
AD     67.189     0.003
```

AE 126.760 0.003

HD BE
BE1 23.393 0.005
BD 104.089 0.003

HD AB
AC 89.285 0.003

HD BB
BC 168.379 0.003
BD 113.531 0.003
BE 215.697 0.003

HD AE
AC 126.757 0.003
AD 96.093 0.003

#(Mottram, 2009)
HD BD
BC 137.606 0.003
BE1 97.257 0.003

SUPPLEMENTARY OBSERVATIONS

#TC405 observations by IIA students, Summer 2009
HA AB AC1 AD 83 52 52 7
HA AE AD AC1 91 49 47 7
HD AC1 AB 98.570 .003
HD AC1 AD 62.178 .003
HD AD AE 96.094 .003

HA CHIM BB BC 22 35 31 7
HA BB BC BD 317 55 45 7
HA BE1 BD BC 261 07 56 7
HA RCSP BE1 BD 283 59 39 7 *
HD BB BC 168.379 .003
HD BC BB 168.379 .003
HD BC BD 137.600 .01 * Disagrees with 2 obs in March 2010
HD BD BC 137.599 .01 * Disagrees with 2 obs in March 2010
HD BE1 BD 97.259 .003
HD BD BE1 97.257 .003

HA RCSP DB DD 57 53 58 7
HA DB DC DD 295 22 39 7
HA DC DD DE 104 36 10 7
HA CHIM DE DD 251 06 30 20
HD DC DB 111.588 .003
HD DB DD 106.823 .003
HD DD DC 83.133 .003
HD DE DD 174.339 .003

```
#Observations for area C averaged from PGA courses, 2000-2009
HA RCSP CB   CC     9 26 02 5
HA CB   CC   CD    86 54 11 5
HA CC   CD   CE   241 42 50 5
HA CD   CE   CHIM 190 14 05 5
HD CB   CC   121.820 .003
HD CC   CD    86.283 .003
HD CD   CE    90.545 .003

#Area D, Dec 2009
HA RCSP DB   DC    13 13 17 5
HA DB   DC   DD   295 22 32 5
HA DC   DD   DE   104 36 07 5
HA DD   DE   CHIM 108 54 01 5
HD DB   DC   111.589 .003
HD DC   DD    83.131 .003
HD DE   DD   174.337 .003

#Observations to fix KCSW by ALJ, 23/2/10
HO AUX
DD    00 00 00  00 00 02  3
DC    50 28 08  50 28 11  3
KCSW  53 00 59  53 01 02  3
DD   359 59 58  00 00 00  3

HD AUX
DD  98.193  .003
DC  96.784  .003

#Student observations, March 2010
HA RCSP AB   AC1  292 33 21 5
HA AB   AC1  AD    83 53 26 20 *Disagrees with summer 2009 obs
HA AC1  AD   AE   268 10 23 5
HA AD   AE   KCSW 179 59 8 7 *disagrees with Dempster/Downes
HD AB   AC1   98.572 .003
HD AC1  AD    62.180 .003
HD AD   AE    96.093 .003

HA CHIM BB   BC   22 35 23 20 *disagrees with Jacobs/Sbiti and Dempster/Downes
HA BB   BC   BD  317 56 23 30 *disagrees with summer 2009 obs
HA BC   BD   BE1  98 51 40 20 *disagrees with obs 4 blocks down
HA BD   BE1  RCSP 76 0 38 5
HD BB   BC   168.385 .003
HD BC   BD   137.609 .003
HD BD   BE1   97.259 .003

HA RCSP CB   CC     9 26 03 5
HA CB   CC   CD    86 53 53 20 *disagrees with obs 4 blocks down
HA CC   CD   CE   241 42 51 5
HA CD   CE   CHIM 190 15 02 60 *disagrees with obs 4 blocks down
HD CB   CC   121.821 .003
```

HD CC   CD   86.288 .003
HD CD   CE   90.547 .003

HA RCSP DB   DC   13 13 16 5
HA DB   DC   DD   295 22 44 5
HA DC   DD   DE   104 36 16 5
HA DD   DE   CHIM 108 54 05 10 *suspect
HD DB   DC   111.590 .003
HD DC   DD   83.135 .003
HD DD   DE   174.340 .003

HA RCSP AB   AC1   292 33 09 5
HA AB   AC1 AD   83 52 51 10 *fights with Dempster/Downes
HA AC1 AD   AE   268 10 20 5
HA AD   AE   KCSW 179 59 12 10 *disagrees with Dempster/Downes
HD AB   AC1   98.570 .003
HD AC1 AD   62.179 .003
HD AD   AE   96.094 .003

HA CHIM BB   BC   22 35 12 20 *needs checking - fights with Jacobs/Sbiti and Dempster/Downes
HA BB   BC   BD   317 56 20 20 *disagrees with summer 2009 obs
HA BC   BD   BE1   98 52 03 5
HA BD   BE1 RCSP 76 00 30 5
HD BB   BC   168.382 .003
HD BC   BD   137.609 .003
HD BD   BE1   97.256 .003

HA RCSP CB   CC   9 26 03 5
HA CB   CC   CD   86 54 14 5
HA CC   CD   CE   241 42 48 5
HA CD   CE   CHIM 190 14 29 20 *needs checking
HD CB   CC   121.817 .003
HD CC   CD   86.287 .003
HD CD   CE   90.547 .003

#Student observations, 17 March 2011
HA RCSP AB   AC   295 57 31 5
HA AB   AC   AD   89 00 05 5
HA AC   AD   AE   259 39 06 5
HA AD   AE   KCSW 179 58 39 10 *disagrees with obs on 18 March
HD AB   AC   89.289 .003
HD AC   AD   67.188 .003
HD AD   AE   96.097 .003

HA CHIM BB   BC   22 35 14 20 *disagrees with obs on 18 March
HA BB   BC   BD 317 55 32 10 *disgrees with previous measurements
HA BC   BD   BE 111 38 25 5   *needs checking
HA BD   BE   RCSP 63 19 41 5
HD BB   BC   168.379 .003
HD BC   BD   137.600 .003
HD BD   BE   104.088 .003

HA RCSP CB   CC   9 26 02 5
HA CB   CC   CD   86 54 02 5
HA CC   CD   CE   241 42 41 5
HA CD   CE   CHIM 190 13 59 5
HD CB   CC   121.820 .003
HD CC   CD   86.285 .003
HD CD   CE   90.546 .003

HA RCSP DB   DC   13 13 14 5
HA DB   DC   DD   295 22 39 5
HA DC   DD   DE   104 36 38 20 *disagrees with obs on 18 March
HA DD   DE   CHIM 108 54 20 20 *disagrees with obs on 18 March
HD DB   DC   111.588 .003
HD DC   DD   83.137 .003
HD DD   DE   174.337 .003

#Student observations, 18 March 2011
HA RCSP AB   AC   295 57 29 5
HA AB   AC   AD   89 00 06 5
HA AC   AD   AE   259 39 10 5
HA AD   AE   KCSW 179 58 55 5
HD AB   AC   89.288 .003
HD AC   AD   67.188 .003
HD AD   AE   96.094 .003

HA CHIM BB   BC   22 35 24 10 *Seems too low as well - check?
HA BB   BC   BD 317 55 45 5
HA BC   BD   BE 111 38 31 10 *disagrees with obs on 17 March
HA BD   BE   RCSP 63 19 43 5
HD BB   BC   168.382 .003
HD BC   BD   137.600 .003
HD BD   BE   104.089 .003

HA RCSP CB   CC   9 26 05 5
HA CB   CC   CD   86 54 04 5
HA CC   CD   CE   241 42 50 5
HA CD   CE   CHIM 190 13 56 5
HD CB   CC   121.820 .003
HD CC   CD   86.284 .003
HD CD   CE   90.546 .003

HA RCSP DB   DC   13 13 18 5
HA DB   DC   DD   295 22 37 5
HA DC   DD   DE   104 36 09 5
HA DD   DE   CHIM 108 54 06 5
HD DB   DC   111.589 .003
HD DC   DD   83.132 .003
HD DD   DE   174.339 .003

#DIGITAL LEVELLING DATA
*(Downes/Dempster, 2011)

```
DH BMY B2 -3.15825 0.005 *Imprecise bracket placement
DH B2  B1  0.04488 0.001
DH B1  AB -0.14920 0.001
DH B1  BB  0.14843 0.001
DH BB  AB -0.29763 0.001

DH AB  A2  0.17167 0.001
DH A2  A3 -0.15133 0.001
DH A3  B2  0.08314 0.001
DH A3  D2  1.64279 0.001
DH D2  C1 -0.46898 0.001
DH C1  B2 -1.09068 0.001

*DH D2 DD -0.68971 0.001  *Why are these readings suspended?
*DH DD DB  0.09466 0.001
*DH DB D1 -0.38313 0.001

DH D1  DC  0.69790 0.001
DH DC  D2  0.28027 0.001
DH B1  BD  0.16157 0.001
DH BD  B3  1.28816 0.001
DH B3  BC -1.51255 0.001
DH BC  B2  0.01809 0.001
DH A2  AD -0.66680 0.001
DH AD  AE -0.12332 0.001
DH AE  A1  0.71362 0.001
DH A1  D1  0.58980 0.001
DH D1 BMZ  1.10040 0.005 *Imprecise bracket placement
DH C1  CE -0.03798 0.001
DH CE  CD -0.27693 0.001
DH CD  CB  0.02354 0.001
DH CB  B3  0.69514 0.001
DH BMZ A1 -1.68940 0.005 *Imprecise bracket placement
DH C1  CC  0.02787 0.001
DH CC  C2  0.10083 0.001
DH C2  CB -0.42056 0.001
DH D2  DE -0.57787 0.001
DH AD  AC  0.21024 0.001
DH AC  A3  0.30651 0.001
DH B3  BE -0.92533 0.001
DH BE  BD -0.36283 0.001
DH BMX C2 -0.43697 0.005 *Imprecise bracket placement N.b. This was produced taking
*the average difference measured in each direction so that a loop was formed to check
*closing error

*Repeated Readings
DH DD DC  0.40755 0.001
DH D2 DD -0.68746 0.001
DH DD DB  0.09297 0.001
DH DB D1 -0.38382 0.001
DH BE BMW 0.48507 0.001
DH BMW B3 0.44026 0.001
```